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Mini Review

Harmonic oscillation picture of the free electron Zitterbewegung in vacuum

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Introduction

As shown e.g. in ref.[1] Zitterbewegung (trembling motion) of the free Dirac electron is generated if transitions between positive and negative energy states occur. Here we treat this effect in a single-mode configuration using a density matrix method. As compared with more elaborate conventional treatments, this method allows an easy estimate of the amplitude of the motion. The result is by predicted spreads of the free electron charge.

The density matrix calculation

We consider solutions of the Dirac equation of the form

$$\tilde{\psi} = \tilde{u}(p) + \tilde{v}(\tilde{p}) \quad (1)$$

with p a momentum in a given space direction. We then define a density matrix by the relation

$$\tilde{p} = \tilde{\psi}^{\dagger} \times \tilde{\psi} \quad (2)$$

where the symbol $\times$ designates a tensor product. The quantity $\tilde{p}$ satisfies the Liouville-von Neumann equation, [2] natural units $(\hbar = 1,\ c = 1)$

$$i\frac{\partial \tilde{p}}{\partial t} = \left[h_D, \tilde{p}\right] \quad (3)$$

involving the commutator with the free-electron Hamiltonian h_D. The functions $\tilde{u}(p)$ and $\tilde{v}(p)$ obey the eigen value equations with $E > 0$.

$$h_D \tilde{u}(p) = E\tilde{u}(p) \quad (4a)$$

$$h_D \tilde{v}(p) = -E\tilde{v}(p) \quad (4b)$$

specifying them as positive and negative energy solutions. Introducing the function of eq. (1) into the density matrix of eq. (2) we write

$$\tilde{p} = \left(\tilde{u}^{\dagger}(p) + \tilde{v}^{\dagger}(p)\right) \times \left(\tilde{u}(p) + \tilde{v}(p)\right) = \tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3 + \tilde{p}_4 \quad (5)$$

Here the commutators $\left[h_D, \tilde{p}_1\right] \equiv \left[h_D, \tilde{u}^{\dagger} \times \tilde{u}\right]$ and $\left[h_D, \tilde{p}_2\right] \equiv \left[h_D, \tilde{v}^{\dagger} \times \tilde{v}\right]$ vanish and we are left with the equations

$$i\frac{\partial}{\partial t}\tilde{p}_3 \equiv \left[h_D, \tilde{u}^{\dagger} \times \tilde{v}\right] = 2E\tilde{u}^{\dagger} \times \tilde{v} \quad (6.1)$$

$$i\frac{\partial}{\partial t}\tilde{p}_4 \equiv \left[h_D, \tilde{v}^{\dagger} \times \tilde{u}\right] = -2E\tilde{v}^{\dagger} \times \tilde{u} \quad (6.2)$$

yielding the solutions

$$\tilde{u}^{\dagger} \times \tilde{v} = \left(u^{\dagger} \times v\right) e^{-i2Et} \quad (7.1)$$

$$\tilde{v}^{\dagger} \times \tilde{u} = \left(v^{\dagger} \times u\right) e^{i2Et} \quad (7.2)$$

For the time-independent factors *u(p), v(p)* we now introduce quantities corresponding to a Lorentz boost in the x^3 direction and more explicitly those given by the following expressions [2]

$$\mathrm{u}(\mathrm{p}) = \begin{pmatrix} \sqrt{E - p^3}\begin{pmatrix}1\\0\end{pmatrix} \\ \sqrt{E + p^3}\begin{pmatrix}1\\0\end{pmatrix} \end{pmatrix} \quad (8.1)$$

$$\mathrm{v}(\mathrm{p}) = \begin{pmatrix} \sqrt{E - p^3}\begin{pmatrix}1\\0\end{pmatrix} \\ -\sqrt{E + p^3}\begin{pmatrix}1\\0\end{pmatrix} \end{pmatrix} \quad (8.2)$$

Working out the tensor products we then find after some algebra and omitting for simplicity the superscript on p^3.

$$\rho_3 = \begin{pmatrix} E - p & 0 & m & 0 \\ 0 & 0 & 0 & 0 \\ -m & 0 & -(E+p) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (9.1)$$

$$\rho_4 = \begin{pmatrix} E-p & 0 & -m & 0 \\ 0 & 0 & 0 & 0 \\ m & 0 & -(E+p) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad 9.2)$$

So far the density matrix has not been normalized to unity. To achieve normalization we first notice that we have $Tr u^{\dagger} \times u = u^{\dagger}u = 2E$ and $Tr v^{\dagger} \times v = v^{\dagger}v = 2E$, from the definitions of these quantities. Noticing further that, according to eq.'s (8.1) and (8.2) we have $Tr\rho_3 = Tr\rho_4 = -2p$, we arrive at the total trace

$$Tr\rho = 4E - 4p = 4(E-p) \qquad (10)$$

Let us now consider the amplitude operator of the oscillatory electron motion, which we define as follows :

$$\tilde{x} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & x^1 & 0 & 0 \\ 0 & 0 & x^2 & 0 \\ 0 & 0 & 0 & x^3 \end{pmatrix} \qquad (11)$$

and calculate the average values $Tr(\rho_3 \hat{x})$ and $(\rho_4 \hat{x})$. Using eq.(8.1) and (8.2) we then find

$$Tr(\rho_3 \hat{x}) = Tr(\rho_4 \hat{x}) = -\frac{1}{4}\frac{E+p}{E-p}x^2 \qquad (12)$$

where the normalization factor of eq.(10) has been taken into account.

To describe the motion of the electron, we then add the time-dependent factors of eq.'s (7.1) and (7.2). and thus consider the quantity

$$Tr(\rho_3 \hat{x})e^{-i2Et} + Tr(\rho_4 \hat{x})e^{i2Et} = -\frac{1}{2}\frac{E+p}{E-p}x^2 cos(2Et) \qquad (13)$$

Discussion

In this approach, according to eq. (13), there are rapid harmonic oscillations in the χ^2 direction i.e. perpendicular to the translatory motion of momentum p^3. To estimate the amplitude of these oscillations, known as Zitterbewegung, we define a velocity by differentiating the r.h.s. of eq.(13) and write.

$$v(t) == -\frac{1}{2}\frac{E+p}{E-p}x^2 \frac{d}{dt}cos(2Et) = \frac{E+p}{E-p}Ex^2 sin(2Et) \qquad (14)$$

with the maximum value

$$v_{max} = \frac{E+p}{E-p}Ex^2 \qquad (15)$$

becoming in the rest frame with $p = 0$

$$v_{max} = mx^2 \qquad (16)$$

Clearly v_{max} cannot exceed c or 1 in natural units, and by adopting this value we obtain from eq.(16) the amplitude

$$x^2 = \frac{1}{m} \qquad (17)$$

equal to the Compton wavelength.

Concluding remarks

The result thus obtained conforms with the idea found in the literature [3,4], according to which there is an apparent spread of the free electron charge over a region with the dimension of the Compton wavelength.

Note however that a different approach to this problem is presented in Weisskopf's positron-hole theory [4].

Note finally that Zitterbewegung need not be attributed to interference between positive and negative energy states if the Dirac theory is reformulated accordingly [5].

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