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Research Article

What is the True Value of Fuzzy Reasoning in the Framework of the 'Human Intelligence' Linguistic Variable?

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Abstract

Computations are presented in the framework of 'human intelligence' linguistic variable. Computation concern fuzzy reasoning i.e. the true value of the implication '*IF...THEN*'. Computation reveal high dependence of the true value of a rule on the meaning of fuzzy logic operator '*IF...THEN*'. The effect of the incorporation of different kind of understanding of the fuzzy logic expression 'Intelligent' in the premises of fuzzy reasoning is presented. This framework makes it possible that in cardiology "severe" and "moderate" pathology may be both be "true "for a given patient.

More Information

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Keywords: IQ index; Linguistic variable; Human intelligence; Fuzzy reasoning

1. Introduction

Definition 1.1

In classical logic reasoning consists of variables (also called arguments), coupled by logical operators forming a logical expression and a corresponding consequence. The variables a_1 *is* A_1, a_2 *is* A_2,..., a_k *is* A_k are statements, which can be false or true. The structure of the logical expression is:

$$(a_1 \text{ is } A_1) \bowtie_1 (a_2 \text{ is } A_2) \bowtie_2 \ldots \bowtie_{k-1} (a_k \text{ is } A_k) \quad (1.1)$$

Where the symbol $\bowtie_i$ is one of the logical operators: *NOT; AND; OR; XOR.* The logical expression is formulated usually with simple uni-and bivariate logical operators and sentences.

The structure of a rule of reasoning in classical logic is:

$$If\ (a_1 \text{ is } A_1) \bowtie_1 (a_2 \text{ is } A_2) \bowtie_2 \ldots \bowtie_{k-1} (a_k \text{ is } A_k) \text{ then } B \quad (1.2)$$

Where the consequence *B* is a statement, which can be false or true.

Definition 1.2

The true value of reasoning in classical logic consist in the expectation that if the logical expression is true, (the conditions of the rule are fulfilled) then the consequence is true.

In classical logic the Boolean calculus [1] assign to any sentence or rule two values: 0 in case of the sentence or rule when this is false and 1 in case of the sentence or rule when this is true. In the following Table 1, the true values are given in case of the application of different logical operators.

Where, *XOR* stands for "either..., or..."

Example 1.1 demonstrates reasoning and Boolean True Value (BTV) computation used in set theory.

Table 1

A	B	Not A	A(AND)B	A(OR)B	A(XOR)B	A(imply)B
1	1	0	1	1	0	1
1	0	0	0	1	1	0
0	1	1	0	1	1	1
0	0	1	0	0	0	1

The results in Table.2, reflect the property of the implication operator that if the logical expression is true, (the conditions of the rule are fulfilled) and the consequence is false then the implication is false. In all the other situations, the implication is true. However, this is not exactly what we expect when we speak about the true value of the implication:

If $(A1 \subseteq A2)$ AND $(u \in A1)$ then $(u \in A2)$. That is because the above implication is what is called in classical logic 'syllogism'.

Syllogism is a "Greek" word that means inference or deduction. A *syllogism* is a form of deductive reasoning in which a conclusion is drawn from two or more premises. In case of two premises, this logical structure consists of three parts: a major premise, a minor premise, and a conclusion. The major premise is a general sentence, the minor premise is a specific sentence, and the conclusion is a sentence which true value logically (i.e. according to the human thinking) is accepted if the two premises are true. For example:

1. All mammals are animals.
2. Camels are mammals.
3. Camels are animals.

As long as premise one and premise two are true, then the conclusion must also be true. If mammals are animals, and camels are mammals; there is no way camels aren't animals!

In Aristotle, each of the premises is in the form "All S are P," "Some S are P", "No S are P" or "Some S are not P", where "S" is the subject-term and "P" is the predicate-term: All S are P," and "No S are P" are termed universal propositions; "Some S are P" and "Some S are not P" are termed particular propositions. The two premises has a term in common, which is called the middle term. In the above example :mammals:=S_1; animals:=P_1;camels:=S_2 ;animals:=P_2 ;middle term:=S_1=P_2 .

The conclusion is a sentence in which: the subject-term is the same with the subject-term S_2 of the minor premise and the predicate-term is the same with the predicate-term P_1ofthe major premise. Each of the premises has one term in common with the conclusion. In the above example, the conclusion structure is S_2 are P_1.

There are infinitely many possible syllogisms, but only 256 logically distinct types and only 24 valid types.

A syllogism in terms of conclusion

All S are P takes, the form:

Major premise: All M are P.

Minor premise: All S are M**.**

Conclusion/Consequent: All S are P**.**

(Note: M – Middle, S – subject, P – predicate.)

Syllogisms are the most common way of arranging premises into a good argument. A deductive argument moves from the general to the specific and opposes inductive arguments that move from the specific to the general.

This connection between the logical expression and conclusion which exist in a syllogism is completely ignored in case of the Boolean True Value computation of the implication if $(A1 \subseteq A2)$ AND $(u \in A1)$ then $(u \in A2)$. The effect of ignorance can be seen changing for example in the above reasoning the conclusion putting a new conclusion for example' camels are animals.' The BTV of the so obtained implication is the same as in the case of Table 1.butwhat we think is that the so called 'true value of reasoning' is different in the two cases. Thus, the computed BTV of the implication is not fully appropriate for the evaluation of the 'true value of the reasoning' because takes into account only on the BTV of conclusion and ignore other kind of connection, which exists between the logical expression and conclusion in case of syllogism, which is a correct and unanimously accepted kind of

> *#example of raisoning and Boolean true value (Btv) computation used in set theory in classical logic;*

$if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ \ then\ (u \in A2).$

$\#A1 := is\ a\ first\ set\ of\ objects;\ A2 := is\ a\ second\ set\ of\ objects;\ u := is\ an\ element;$

$\#(A1 \subseteq A2)$ *is the statement : any element of the set A1 belongs* ***to*** *the set A2;*

> $\#(u \in A1)$ *is the statement : element u is element of the set A1;*

> $\#(u \in A2)$ *is the statement : element u is element of the set A2;*

> *#computation of the Boolean true value (Btv) of the logical expression* $(A1 \subseteq A2)\ AND\ (u \in A1)$

> $Btv(A1 \subseteq A2) = 1,\ Btv(u \in A1) = 1,\ Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 1$

> $Btv(A1 \subseteq A2) = 1,\ Btv(u \in A1) = 0,\ Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0$

> $Btv(A1 \subseteq A2) = 0,\ Btv(u \in A1) = 1,\ Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0$

> $Btv(A1 \subseteq A2) = 0,\ Btv(u \in A1) = 0,\ Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0$

> *#computation of the Boolean true value of the conclusion* $(u \in A2)$

> $Btv(u \in A2) = 1 > Btv(u \in A2) = 0$

> *#computation of the Boolean true value of the rule :* $if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2).$

> $Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 1,\ Btv(u \in A2) = 1,\ Btv[\,if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2)\,] = 1$

> $Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0,\ Btv(u \in A2) = 1,\ Btv[\,if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2)\,] = 1$

> $Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0,\ Btv(u \in A2) = 1,\ Btv[\,if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2)\,] = 1$

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> $Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0,\ Btv(u \in A2) = 0,\ Btv[\,if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2)\,] = 1$

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> $Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 0,\ Btv(u \in A2) = 0,\ Btv[\,if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2)\,] = 1$

Remark that in the above table only the Boolean true value of the implication is equal to zero.

$Btv[\,(A1 \subseteq A2)\ AND\ (u \in A1)\,] = 1,\ Btv(u \in A2) = 0,\ Btv[\,if\ (A1 \subseteq A2)\ AND\ (u \in A1)\ then\ (u \in A2)\,] = 0$

Table 2

reasoning in human thinking. Obviously, the 'true value of a reasoning' has to depend on the arguments to which the reasoning has to be applied but its dependence on conclusion is not so 'weak' as in the case of implication is described.

2. Conditional possibility and fuzzy logic operator *'IF… THEN'* in the framework of the 'human intelligence' linguistic variable

According to [2], using the conditional possibility distribution, in fuzzy logic, two type of logic operator *'IF ... THEN'* can be defined.The so called

minimum fuzzy logic operator 'IF ... THEN' and a so called

product fuzzy logic operator 'IF...THEN'

Definition 2.1

Let A a fuzzy subset of universe U and B a fuzzy subset of universe V respectively and two fuzzy statements (u is A), (v is B).

The minimum fuzzy logic operator 'IF ... THEN' Transform these fuzzy statements into the fuzzy statement denoted usually by *minimum'IF* (u *is* A)*THEN*(v is B)'. The fuzzy subset $C_{minimum\ IF\ ...\ THEN,}$ representing the fuzzy statement *minimum'IF*(u *is* A)*THEN* (v is B)', is a subset of the universe $U \times V$ andaccording to [2] its membership function is

$$f_{C_{minimum\,IF\ldots THEN}}(u,v) = 1 \text{ for } f_A(u) < f_B(v) \text{ and } f_{C_{minimum\,IF\ldots THEN}}(u,v) = f_B(v) \text{ for } f_A(u) > f_B(v) \text{ for } f_A(u) > f_B(v). \quad (2.1)$$

The 'true value' or 'degree of fulfillment' of the fuzzy statement '*minimum'IF* (u *is* A)*THEN*(v is B)'. ' denoted by *DOF (minimum'IF* (u *is* A)*THEN*(v is B)' is given by:

$$DOF\left(minimum' IF\,(u\,is\,A)THEN\,(v\,is\,B)'\right) = 1 \;\; for\; f_A(u) < f_B(v)$$

and

$$DOF\left(minimum' IF\,(u\,is\,A)THEN\,(v\,is\,B)'\right) = f_B(v) \; for\; f_A(u) > f_B(v) \quad (2.2)$$

If A is the fuzzy subset of very intelligent persons $A = A_{very\ intelligent}$ and B is the fuzzy subset of intelligent persons $B = B_{intelligent}$ then remember first that $f_{A_{very-intelligent}}(x) = f^2_{A_{intelligent}}(x)$ see [2].

In the following we analyze the computed dependence of 'true value' of the fuzzy statement *minimum'IF* (u *is* A)*THEN*(v is B)' on the couple (u,v) in the framework of 'human intelligence'linguistic variable.

At the beginning consider the case $40 < u \le 100$ and $40 < v \le 100$. Computing the membership value of function $f_{C_{minimum\,IF\ldots THEN}}$ of the statement

$minimum' IF\left(u\,is\,A_{very\,intelligent}\right)THEN\left(vis\;\; B_{intelligent}\right)'$ for $u = 53$ and $v = 81$; The following result is found:

$$f_{A_{very\,intelligent}}(53) = 0.04694444444,\; f_{B_{intelligent}}(81) = 0.6833333333.$$

Hence $f_{A_{very\,intelligent}}(53) < f_{B_{intelligent}}(81).$. Therefore $f_{C_{minimum\,IF\ldots THEN}}(53,81) = 1.$ More generally for $40 < u \le 100$ and

$40 < \frac{u^2}{60} - \frac{4 \times u}{3} + \frac{200}{3} < v \le 100$ we find $f_{A_{very\,intelligent}}(u) < f_{B_{intelligent}}(v).$

Therefore $f_{C_{minimum\,IF\ldots THEN}}(u,v) = 1.$

The above computation reveal a whole region of couples (u, v), where the minimum possible true value of the implication $minimum' IF(u\,is\,A_{very\,intelligent})THEN(vis\;\; B_{intelligent})'$ is equal to 1.

For $u = 81$ $f_{A_{very\,intelligent}}(81) = 0.4669444444$ and for $v = 53$ $f_{A_{intelligent}}(53) = 0.2166666667$.

Hence $f_{B_{very\,intelligent}}(53) > f_{A_{intelligent}}(81)$.

Therefore $f_{C_{minimum\,IF\ldots THEN}}(81,53) = 0.2166666667$.

More generally for 40 < u ≤ 100 and $40 < v < \frac{u^2}{60} - \frac{4 \times u}{3} + \frac{200}{3} \le 100$ we find $f_{A_{very\,intelligent}}(u) > f_{B_{intelligent}}(v)$.

Therefore $f_{C_{minimum\,IF\ldots THEN}}(u,v) = \frac{v-40}{60}$.

This computation reveals a region of couples (u, v), where the minimum possible true value of the implication $minimum'\,IF(u\,is\,A_{very\,intelligent})THEN(v\,is\;\;B_{intelligent})'$ is $\frac{v-40}{60} =$ the conclusion true value $= f_{B_{intelligent}}(v)$.

This fact is surprising because in classical understanding of the word very intelligent always imply intelligent. Therefore our expectation is that the true value is equal to 1.The explanation is that, in the classical understanding, in the mind there is an implicit hypothesis; this is that both faculties 'intelligent' and 'very intelligent' concern the same person. But in the present discussion the degree of confidence in case of very intelligent person $f_{A_{very\,intelligent}}(u)$ is not necessarily the same as that of the intelligent person $f_{B_{intelligent}}(v)$. For this reason there is a borderline in the 'space' of parameters (u, v).

The borderline split the 'space' of parameters $(u,v) \in [40,100] \times [40,100]$ in two regions. In the region where the confidence degree of the statement 'very intelligent' i.e $f_{A_{very\,intelligent}}(u)$. is less than the confidence degree of the conclusion statement 'intelligent' i.e. $f_{B_{intelligent}}(v)$, the minimum possible true value of the implication is equal to 1. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\,intelligent}}(u)$ is more than the confidence degree of the statement 'intelligent', $f_{B_{intelligent}}(v)$, the minimum possible true value of the implication is $\frac{v-40}{60} = f_{B_{intelligent}}(v)$.

The above-delimited regions can be seen in the next (Figure 2.1).

In the region situated upper the blue borderline the 'minimum possible true value' of implication is equal to 1.

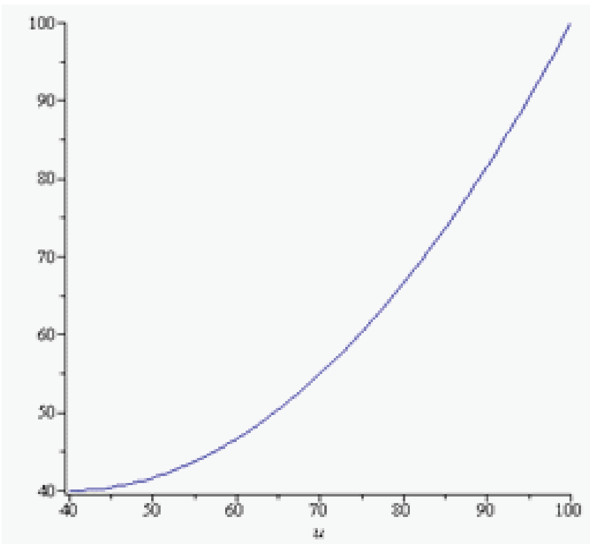

Figure 2.1: Regions : $40 < u \le 100$ and $40 < \frac{u^2}{60} - \frac{4 \times u}{3} + \frac{200}{3} < v \le 100; 40 < u \le 100$ *and* $40 < v < \frac{u^2}{60} - \frac{4 \times u}{3} + \frac{200}{3} \le 100$

In the region situated under the blue borderline the minimum possible true value of the implication is equal to $\frac{v-40}{60}$.

The next step is the analysis of case 100< u < 160 and 40 < v < 100.

For $u = 120\, f_{A_{very\, intelligent}}(120) = 0.4444444444$ and for $v = 53$ $f_{B_{intelligent}}(53) = 0.2166666667$.

Hence $f_{A_{very\, intelligent}}(120) > f_{B_{intelligent}}(53)$ and $f_{C_{minimum\, IF\ldots THEN}}(120,53) = 0.2166666667$

More generally for 100 < u ≤ 160 and $40 < v < \frac{u^2}{60} - \frac{16\times u}{3} + \frac{1400}{3} \le 100$ we find $f_{A_{very\, intelligent}}(u) > f_{B_{intelligent}}(v)$.

Therefore $f_{C_{minimum\, IF\ldots THEN}}(u,v) = \frac{v-100}{60}$.

For $u = 120$ $f_{A_{very\, intelligent}}(120) = 0.4444444444$ and for $v = 81$ $f_{B_{intelligent}}(81) = 0.6833333333$.

Hence $f_{A_{very\, intelligent}}(120) < f_{A_{intelligent}}(81)$ and $f_{C_{minimum\, IF\ldots THEN}}(120,81) = 1$

More generally for 00 < u ≤ 160 and $40 < \frac{u^2}{60} - \frac{16\times u}{3} + \frac{1400}{3} < v \le 100$ we find $f_{A_{very\, intelligent}}(u) < f_{B_{intelligent}}(v)$.

Therefore $f_{C_{minimum\, IF\ldots THEN}}(u,v) = 1$

Also in this case there is a borderline in the 'space' of parameters (u, v).

The borderline splits the parameter space $(u,v) \in [100,160] \times [40,100]$ into two regions. In the region where the confidence degree of the statement 'very intelligent'i.e. $f_{A_{very\, intelligent}}(u)$, is less than the confidence degree of the conclusion statement 'intelligent' i.e. $f_{B_{intelligent}}(v)$, the minimum possible true value of the implication is equal to 1. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\, intelligent}}(u)$ is more than the confidence degree of the statement 'intelligent', $f_{B_{intelligent}}(v)$, the minimum possible true value of the implication is $\frac{v-40}{60} = f_{B_{intelligent}}(v)$.

These defined regions are illustrated in the following (Figure 2.2).

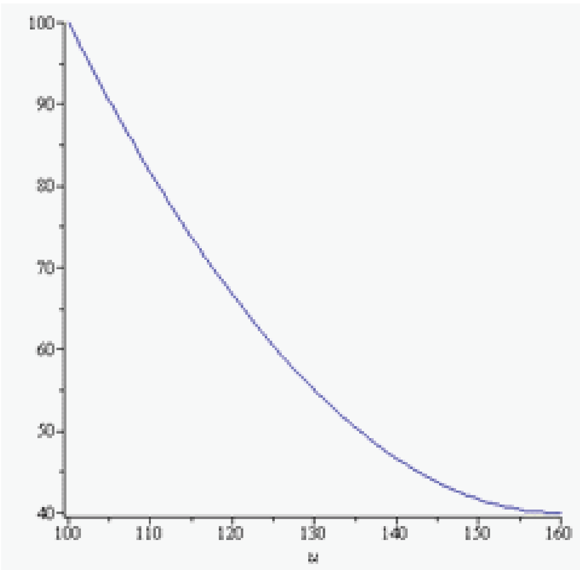

Figure 2.2: Regions : $100 < u \le 160$ and $40 < v < \frac{u^2}{60} - \frac{16\times u}{3} + \frac{1400}{3} \le 100; 100 < u \le 160$ *and* $40 < \frac{u^2}{60} - \frac{16\times u}{3} + \frac{1400}{3} < v \le 100$

If the confidence parameters are in the region situated under the blue borderline then the 'minimum possible true value' of the implication is equal to $\frac{v-40}{60}$ and

If the parameters are in the region above the blue borderline then the 'minimum possible true value' of the implication is equal to 1.

We continue the analysis in the case 40 < u < 100 and 100 < v < 160

For $u=81\, f_{A_{very\, intelligent}}(81)=0.4669444444$ and $v=120\, f_{A_{intelligent}}(120)=0.6666666667.$

Hence $f_{A_{very\, intelligent}}(81)<f_{A_{intelligent}}(120)$ and $f_{C_{minimum\, IF\ldots THEN}}(81,120)=1$

More generally for 40 < u ≤ 100 and $100<v<-\frac{u^2}{60}+\frac{4\times u}{3}+\frac{400}{3}\le 160$ we find $f_{A_{very\, intelligent}}(u)<f_{A_{intelligent}}(v).$

Therefore $f_{C_{minimum\, IF\ldots THEN}}(u,v)=1$

For $u=81\, f_{A_{very\, intelligent}}(81)=0.4669444444$ and for $v=140\, f_{A_{intelligent}}(140)=0.3333333333.$

Hence $f_{A_{very\, intelligent}}(81)>f_{A_{intelligent}}(140)$ and $f_{C_{minimum\, IF\ldots THEN}}(81,140)=0.3333333333.$

More generally for 40 < u ≤ 100 and $40<-\frac{u^2}{60}+\frac{4\times u}{3}+\frac{400}{3}<v\le 160$ we find $f_{A_{very\, intelligent}}(u)>f_{A_{intelligent}}(v).$
Therefore $f_{C_{minimum\, IF\ldots THEN}}(u,v)=\frac{160-v}{60}$

The above delimited regions can be seen in the next (Figure 2.3).

The borderline blue split the 'space' of parameters $(u,\ v)\in[40,100]\times[100,160]$ in two regions.

If a couple (u, v) is in the region situated under the borderline then the minimum possible true value of the implication is equal to 1,and

If a couple (u, v) is in the region situated upper the blue borderline then the 'minimum possible true value' of the implication is equal to $\frac{160-v}{60}$.

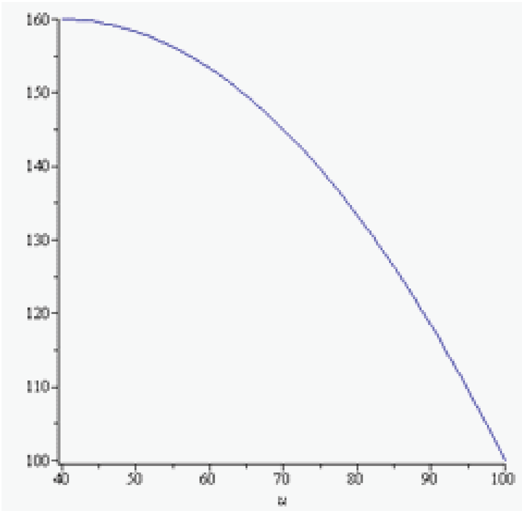

Figure 2.3: $\text{Regions}: 40<u\le 100 \text{ and } 100<v<-\frac{u^2}{60}-\frac{4\times u}{3}+\frac{400}{3}\le 160; 40<u\le 100 \text{ and } 40<-\frac{u^2}{60}+\frac{4\times u}{3}+\frac{400}{3}<v\le 160$

The next step is the analysis of case 100 < u < 160 and 100 < v < 160

For $u = 135\ f_{A_{very\,intelligent}}(135) = 0.1736111111$ and for $v = 120\ f_{A_{intelligent}}(120) = 0.6666666667$.

Hence $f_{A_{very\,intelligent}}(135) < f_{A_{intelligent}}(120)$ and $f_{C_{minimum\,IF\ldots THEN}}(135,120) = 1$

More generally for 100 < u ≤ 160 and $100 < v < -\frac{u^2}{60} + \frac{16 \times u}{3} + \frac{800}{3} \le 160$ we find

$f_{A_{very\,intelligent}}(u) < f_{A_{intelligent}}(v)$. Therefore $f_{C_{minimum\,IF\ldots THEN}}(u,v) = 1$

For $u = 135\ f_{A_{very\,intelligent}}(135) = 0.1736111111$ and for $v = 150\ f_{A_{intelligent}}(150) = 0.1666666667$.

Hence $f_{A_{very\,intelligent}}(135) > f_{A_{intelligent}}(150)$ and $f_{C_{minimum\,IF\ldots THEN}}(135,150) = 0.1666666667$.

More generally for $100 < u \le 160$ and $100 < -\frac{u^2}{60} + \frac{16 \times u}{3} - \frac{800}{3} < v \le 160$ we find $f_{A_{very\,intelligent}}(u) > f_{A_{intelligent}}(v)$. Therefore $f_{C_{minimum\,IF\ldots THEN}}(u,v) = \frac{160 - v}{60}$

The above-delimited regions can be seen in the next (Figure 2.4).

The blue borderline split the 'space' of parameters (u, v) ∈ [100,160]×[100,160] in two regions.

If a couple (u, v) is in the region situated under the borderline then the minimum possible true value of the implication is equal to 1 and

If a couple (u, v) is in the region situated upper the blue borderline then the 'minimum possible true value' of the implication is equal to $\frac{160 - v}{60}$.

The next (Figure 2.5) summarize the 'minimum possible true value of the implication $minimum'\,IF\,(u\,is\,A_{very\,intelligent})\,THEN\,(v\,is\ B_{intelligent})'$ for any couple $(u,\ v) \in [40,160] \times [40,160]$

Definition 2.2

Let A a fuzzy subset of universe U and B a fuzzy subset of universe V respectively and two fuzzy statements (u is A), (v is B).

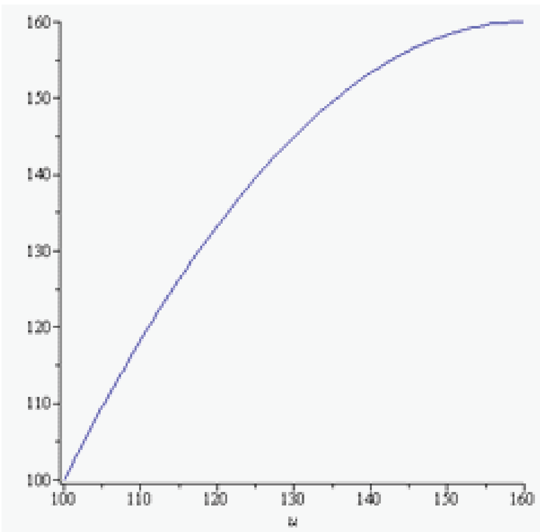

Figure 2.4: Regions : $100 < u \le 160$ and $100 < v < -\frac{u^2}{60} - \frac{16 \times u}{3} - \frac{800}{3} \le 160; 100 < u \le 160$ *and* $100 < -\frac{u^2}{60} + \frac{16 \times u}{3} - \frac{800}{3} < v \le 160$

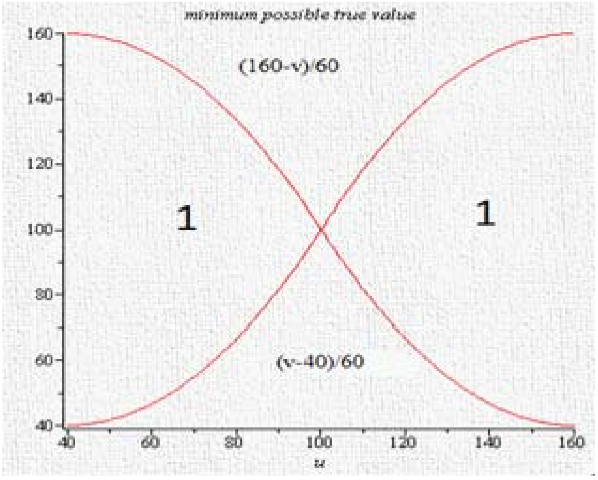

Figure 2.5: Represent the computed dependence, in the framework of 'human intelligence' linguistic variable, of the minimum possible true value of the implication $minimum' IF(u\,is\,A_{very\,intelligent})THEN(vis\ \ B_{intelligent})'$ on the couple (u, v) ∈ [40,160] × [40,160].

The product fuzzy logic operator 'IF ... THEN' Transform these fuzzy statements into the fuzzy statement denoted usually by $product' IF(u\,is\,A)THEN(vis\,B)'$. The fuzzy subset $C_{product\,IF...THEN}$, representing the fuzzy statement $duct' IF(u\,is\,A)THEN(vis\,B)'$, is a subset of the universe $U \times V$ and according to [2] its membership function is

$$f_{C_{product\,IF...THEN}}(u,v)=1 \text{ For } f_A(u)=0 \text{ and } f_{C_{product\,IF...THEN}}(u,v)=minimum\{1,\frac{f_B(v)}{f_A(u)} f_B(v) \text{ for } f_A(u)>0\}. \quad (2.3)$$

The 'product possible true value' or 'product possible degree of fulfillment' of the fuzzy statement $product\ 'IF(u\,is\,A)THEN(v\,is\,B)'$ denoted, $DOF(product\ 'IF(u\,is\,A)THEN(vis\,B)')$

is given by:

$$DOF\left(product\ 'IF(u\,is\,A)THEN(vis\,B)'\right)=1 \ \ for\ f_A(u)=0 \ \text{ and}$$
$$DOF\left(product\ 'IF(u\,is\,A)THEN(vis\,B)'\right)=minimum\left\{1,\frac{f_B(v)}{f_A(u)}\right\} f_A(u)>0. \quad (2.4)$$

If A is the fuzzy subset of very intelligent persons $A = A_{very\ intelligent}$ and B is the fuzzy subset of intelligent persons $B = B_{intelligent}$ then remember first that $f_{A_{very-intelligent}}(x)= f^2_{A_{intelligent}}(x)$ see [2].

In the following we analyze the computed dependence of 'true value' of the fuzzy statement' $product\ 'IF(u\,is\,A)THEN(vis\,B)'$ on the couple (u,v) in the framework of 'human intelligence' linguistic variable.

Computing the membership function $f_{C_{product\,IF...THEN}}$ of the fuzzy statement

$product' IF(u\,is\,A_{very\,intelligent})THEN(vis\ \ A_{intelligent})'$ the following result is found:

For u = 40, or u = 160 and arbitrary v we have we have $f_{C_{product\,IF...THEN}}(u,v)=1;$

For 40 < u < 100 and 40 < v < 100:

$$\text{if } \frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2}<1\,then f_{C_{product\,IF...THEN}}(u,v)=\min\left\{1,\frac{\frac{v-40}{60}}{\left(\frac{u-40}{60}\right)^2}\right\}=\frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2} \text{ and}$$

$$\text{if } \frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2}>1\,then\ f_{C_{product\,IF...THEN}}(u,v)=\min\left\{1,\frac{\frac{v-40}{60}}{\left(\frac{u-40}{60}\right)^2}\right\}=1$$

Therefore:

if u=60 and v=70 , then $\frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2} = \frac{9}{2} > 1$ and $f_{C_{product\ IF\ldots THEN}}(u,v) = \min\left\{1, \frac{9}{2}\right\} = 1;$

if u=95 and v=50 then $\frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2} = \frac{24}{121} < 1$ and $f_{C_{product\ IF\ldots THEN}}(u,v) = min\left\{1, \frac{24}{121}\right\} = \frac{24}{121}$

More generally for $40 < u \le 100$ and $40 < v \le 100$ the blue borderline, presented in the next (Figure 2.6), and defined by the equation

$$v = \frac{1}{60} \times u^2 - \frac{4}{3} \times u + \frac{200}{3} \qquad (2.5)$$

Split the 'space' of parameters $(u,v) \in [40,100] \times [40,100]$ into two regions. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\ intelligent}}(u) = (\frac{u-40}{60})^2$ is less than the confidence degree of the conclusion statement 'intelligent' i.e. $f_{B_{intelligent}}(v) = \frac{v-40}{60}$, the possible product true value of the implication is equal to 1. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\ intelligent}}(u) = (\frac{u-40}{60})^2$ is more than the confidence degree of the statement 'intelligent' i.e. $f_{B_{intelligent}}(v) = \frac{v-40}{60}$, the product possible true value of the implication is $\frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2}$.

These regions can be seen on the next (Figure 2.6).

In the region situated upper the blue borderline the 'possible product true value' of implication is equal to 1.

In the region situated under the blue borderline the 'possible product true value' of the implication is equal to $\frac{\frac{v-40}{60}}{(\frac{u-40}{60})^2}$.

For $100 < u \le 160$ and $40 < v \le 100$ the blue borderline, presented in the next (Figure 2.7), and defined by the equation

$$v = \frac{1}{60} \times u^2 - \frac{16}{3} \times u + \frac{1400}{3} \qquad (2.6)$$

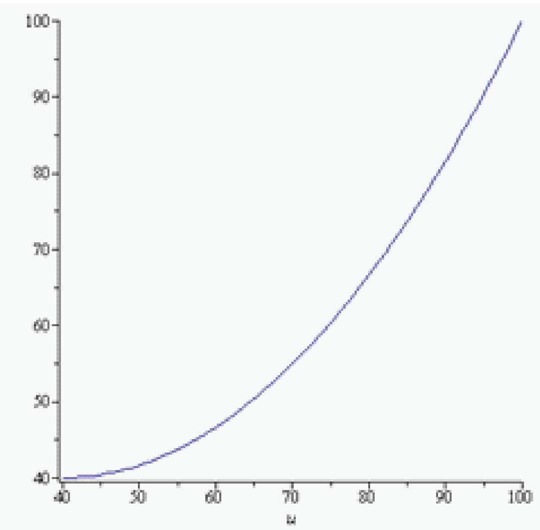

Figure 2.6: Regions : $40 < u \le 100$ and $40 < \frac{u^2}{60} - \frac{4 \times u}{3} + \frac{200}{3} < v \le 100;\ 40 < u \le 100$ and $40 < v < \frac{u^2}{60} - \frac{4 \times u}{3} + \frac{200}{3} \le 100$

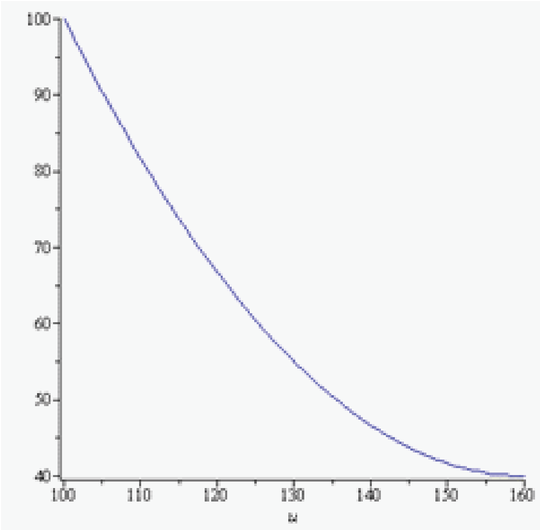

Figure 2.7: Regions: $100 < u \le 160$ and $40 < \frac{u^2}{60} - \frac{16\times u}{3} + \frac{1400}{3} < v \le 100$; $100 < u \le 160$ and $40 < v < \frac{u^2}{60} - \frac{16\times u}{3} + \frac{1400}{3} \le 100$

Split the 'space' of parameters $(u,v)\in[40,160]\times[40,100]$ into two regions. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\,intelligent}}(u) = (\frac{160-u}{60})^2$ is less than the confidence degree of the conclusion statement 'intelligent' i.e. $f_{B_{intelligent}}(v) = \frac{v-40}{60}$, the product possible true value of the implication is equal to 1. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\,intelligent}}(u) = (\frac{160-u}{60})^2$ is more than the confidence degree of the statement 'intelligent' i.e. $f_{B_{intelligent}}(v) = \frac{v-40}{60}$, the product possible true value of the implication is $\frac{\frac{v-40}{60}}{(\frac{160-u}{60})^2}$.

In the region situated upper the blue borderline the 'product possible true value' of implication is equal to 1.

In the region situated under the blue borderline the product possible true value of the implication is equal to $\frac{\frac{v-40}{60}}{(\frac{160-u}{60})^2}$.

For 40 < u ≤ 100 and 100 < v ≤ 160 the blue borderline, presented in the next (Figure 2.8), and defined by the equation

$$v = -\frac{1}{60}\times u^2 + \frac{4}{3}\times u + \frac{400}{3} \tag{2.7}$$

Split the 'space' of parameters $(u,v)\in[40,100]\times[100,160]$ into two regions. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\,intelligent}}(u) = (\frac{u-40}{60})^2$ is less than the confidence degree of the conclusion statement 'intelligent' i.e. $f_{B_{intelligent}}(v) = \frac{160-v}{60}$, the product possible true value of the implication is equal to 1. In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\,intelligent}}(u) = (\frac{40-u}{60})^2$ is more than the confidence degree of the statement 'intelligent' i.e. $f_{B_{intelligent}}(v) = \frac{160-v}{60}$, the product possible true value of the implication is $\frac{\frac{160-v}{60}}{(\frac{40-u}{60})^2}$.

In the region situated upper the blue borderline the 'product possible true value' of implication is equal to $\frac{\frac{160-v}{60}}{(\frac{40-u}{60})^2}$.

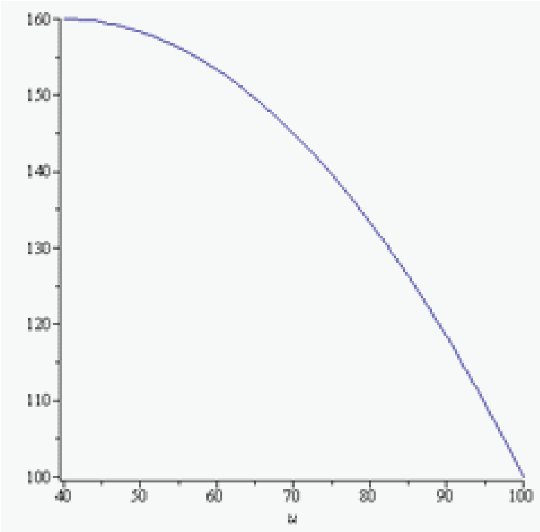

Figure 2.8: Regions : $40 < u \le 100$ and $40 < -\frac{u^2}{60} + \frac{4 \times u}{3} + \frac{400}{3} < v \le 160$; $40 < u \le 100$ and $100 < v < -\frac{u^2}{60} + \frac{4 \times u}{3} + \frac{400}{3} \le 160$

In the region situated under the blue borderline the product possible true value of the implication is equal to 1**.**

For 100 < u ≤ 160 and 100 < v ≤ 160 the blue borderline, presented in the next (Figure 2.9), and defined by the equation

$$v = -\frac{1}{60} \times u^2 + \frac{16}{3} \times u - \frac{800}{3} \tag{2.8}$$

Split the 'space' of parameters $(u, v) \in [100,160] \times [100,160]$ into two regions.

In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\, intelligent}}(u) = (\frac{160-u}{60})^2$ is less than the confidence degree of the conclusion statement 'intelligent' i.e., $f_{B_{intelligent}}(v) = \frac{160-v}{60}$, the product possible true value of the implication is equal to 1.

In the region where the confidence degree of the statement 'very intelligent' i.e. $f_{A_{very\, intelligent}}(u) = (\frac{160-u}{60})^2$ is more than

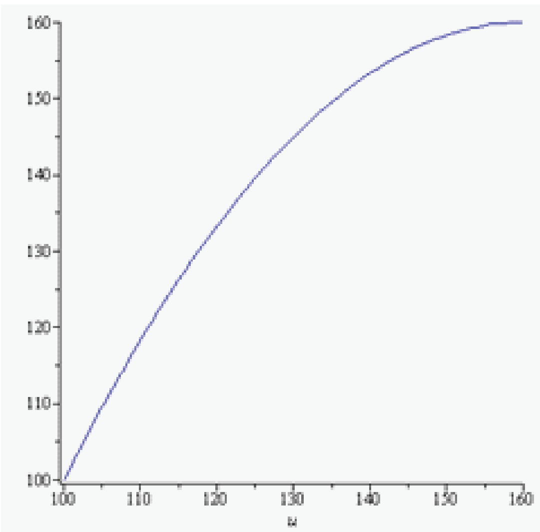

Figure 2.9: Regions : $100 < u \le 160$ and $100 < -\frac{u^2}{60} + \frac{16 \times u}{3} - \frac{800}{3} < v \le 160$; $100 < u \le 160$ and $100 < v < -\frac{u^2}{60} + \frac{16 \times u}{3} - \frac{800}{3} \le 160$

the confidence degree of the statement 'intelligent' i.e. $f_{B_{intelligent}}(v)=\frac{160-v}{60}$, the product possible true value of the implication is $\frac{\frac{160-v}{60}}{(\frac{160-u}{60})^2}$.

In the region situated upper the blue borderline the 'product possible true value' of implication is equal to $\frac{\frac{160-v}{60}}{(\frac{160-u}{60})^2}$.

In the region situated under the blue borderline the product possible true value of the implication is equal to 1**.**

The next (Figure 2.10) summarize the 'product possible true value of the implication $product' IF(u\ is\ A_{very\ intelligent})THEN(v is\ \ B_{intelligent})'$ for any couple $(u,v)\in[40,160]\times[40,160]$.

Comparing Figure 2.10. with Figure 2.5 it can be seen that in the whole region of the couple $(u,v)\in[40,160]\times[40,160]$ the true values of implications $\frac{'product\ IF\left(u\ is\ A_{very\ intelligent}\right)THEN\left(v is\ \ B_{intelligent}\right)'}{'minimum\ \ IF\left(u\ is\ A_{very\ intelligent}\right)THEN\left(v is\ \ B_{intelligent}\right)'}$ are different

3. What means fuzzy logic reasoning in the framework of 'human intelligence' linguistic variable? Can represent the true value of a fuzzy logic expression the true value of the fuzzy logic reasoning?

Definition 3.1

In fuzzy logic, reasoning is a rule, which consists of a set of fuzzy variables (called by some people arguments), coupled by fuzzy logic operators forming a fuzzy logic expression and a corresponding fuzzy consequence (Figure 3.1).

The fuzzy variables $a_1 is\ A_1\ ,a_2 is\ A_2,\ldots,a_k is\ A_k$ are fuzzy statements. $a_1,a_2,\ldots,a_k$, are premises. $A_1,A_2,\ldots A_k$, are fuzzy sets with membership functions $f_{A_1},f_{A_2},\ldots,f_{A_k}$.

The structure of a fuzzy logic expression is:

$(a_1\ is\ A_1)\ \bowtie_1\ (a_2\ is\ A_2)\ \bowtie_2\ \ldots\ \bowtie_{k-1}(a_k\ is\ A_k)$ (3.1)

Where the symbol $\bowtie_i$ is one of the fuzzy logic operators: *NOT; AND; OR; XOR.*

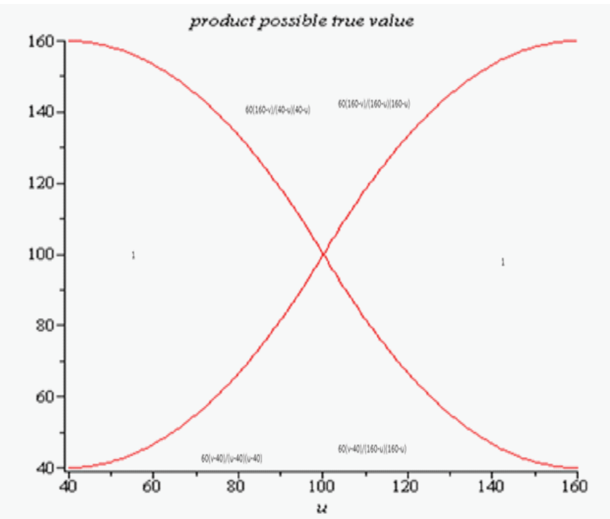

Figure 2.10: Represent the computed dependence, in the framework of 'human intelligence' linguistic variable, of the product possible true value of the implication $'product\ IF(u\ is\ A_{very\ intelligent})THEN(v is\ \ B_{intelligent})'$, On the couple (u, v) ∈[40,160]×[40,160].

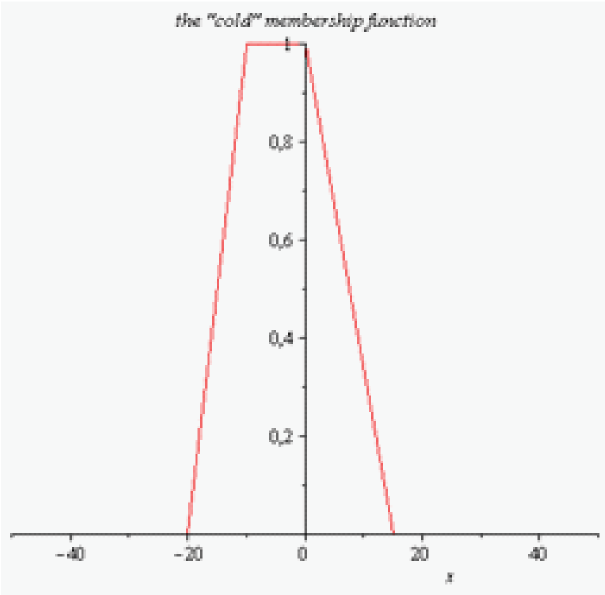

Figure 3.1: The word 'cold' membership function.

The structure of a fuzzy logic rule of reasoning is:

$$If\ (a_1\ is\ A_1) \bowtie_1 (a_2\ is\ A_2) \bowtie_2 \dots \bowtie_{k-1} (a_k\ is\ A_k)\ then\ B \tag{3.2}$$

Instead of the classic case, when the Btv of the logic expression is computed with certainty, in fuzzy logic only the computation of 'degree of fulfillment' (*DOF*) of a fuzzy logic expression for given premises is possible.

The "truth value" or "truth grade" of a fuzzy logic expression is a degree to which the logic expression can be applied to a particular case. On the other hand according to [2] we have:

Definition 3.2

The truth-value corresponding to the fulfillment of the conditions of a fuzzy logic expression for given premises $a_1, a_2, \dots a_k$ is called the Degree of Fulfillment (*DOF*) of that fuzzy logic reasoning. [2] Pg.47.Definition 3.1.

In the following we analyze if the above definition is correct?

Verbal reasoning is often translated into fuzzy logic reasoning. The next example show by computation made in the framework of the 'human intelligence' linguistic variable, that the above definition is questionable (Figure 3.2).

Example.3.1.If it is cold and I have a long way to walk, then I usually take my coat. [2] Example 3.1.pg.45.

Here the fuzzy set A_1 represents the temperature. The word 'Cold' might be characterized with a membership 1 for $-10C \le T \le 0C$, $0\ \ for\, T \ge 15C\, and\, T \le -20\ \ and\ linear\ between.$ This is the trapezoidal fuzzy set (-20, -10, 0, 15) which membership function is: $f_{A_1}(x) = 0\ for\ x < -20;\ f_{A_1}(a_1) = \frac{a_1 + 20}{10}\ for$

$$-20 \le a_1 < -10;\ f_{A_1}(a_1) = 1\ for -10 \le a_1 \le 0;\ f_{A_1}(x) = \frac{15 - a_1}{15}$$
$$for\ 0 < a_1 \le 15;\ f_{A_1}(a_1) = 0\ \ for\, 15 < a_1;$$

The fuzzy word 'long walk' A_2 can be characterized by the triangular fuzzy set (200,1500,4000) meters which membership function is:

$$f_{A_2}(a_2) = 0\ \ for\ \ a_2 < 200\ ;\ f_{A_2}(a_2) = \frac{a_2 - 200}{1300}\ \ for\ \ 200 \le a_2 < 1500\ ;$$
$$f_{A_2}(a_2) = \frac{4000 - a_2}{2500}\ \ for\ \ 1500 \le a_2 < 4000\ ;\ f_{A_2}(a_2) = 0\ \ for\ \ 4000 \le a_2$$

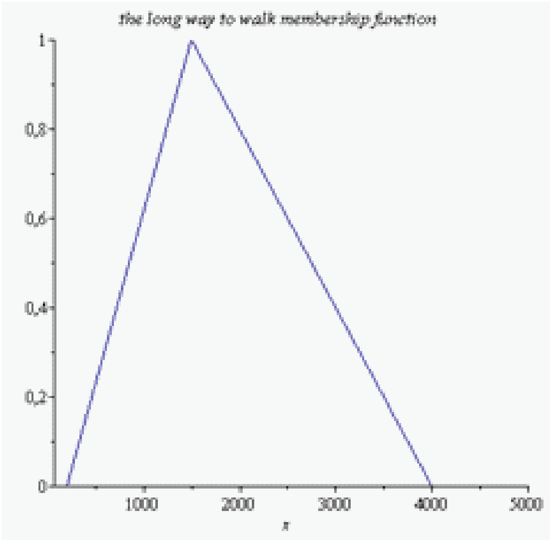

Figure 3.2: The word 'long way to walk' membership function.

The fuzzy word 'usually I take my coat' might be characterized by the triangular fuzzy set $B(0,0.5,1)$ which membership function is:

$$f_B(b) = 0 \ \ for\, b < 0\ ;\ f_B(b) = 2 \times b \ \ \ for\, 0 \le b \le 0.5\ ;$$

$$f_B(b) = -2 \times b + 2 \ \ for\ 0.5 \le b \le 1\ ;\ f_B(b) = 0 \ \ for\, b > 0$$

In this example the fuzzy logic expression is $(a_1 is\, A_1)\, AND\ (a_2 is\, A_2)$. (Figure 3.3)

According to [2] pg.48.in fuzzy logic, there are two type of fuzzy logic operator: the 'minimum fuzzy logic operator *AND*' denoted by $\bowtie_{minimum\text{-}intersection}$, and the 'product fuzzy logic operator *AND*' denoted by $\bowtie_{product\text{-}intersection}$.

Definition 3.3

The 'minimum fuzzy logic operator *AND* transform the fuzzy logic expression ((a_1 is A_1) *AND* [(a_2 is A_2) in the fuzzy statement denoted usually by [(a_1 is A_1) $\bowtie_{minimum\text{-}intersection}$, ($a_2$ is A_2)], and represented by the fuzzy subset $C_{minimum\text{-}intersection}$, which membership function is

$$f_{C_{minimum-intersection}}(a_1, a_2) = \text{minimum}[\, f_{A_1}(a_1), f_{A_2}(a_2)]. \tag{3.3}$$

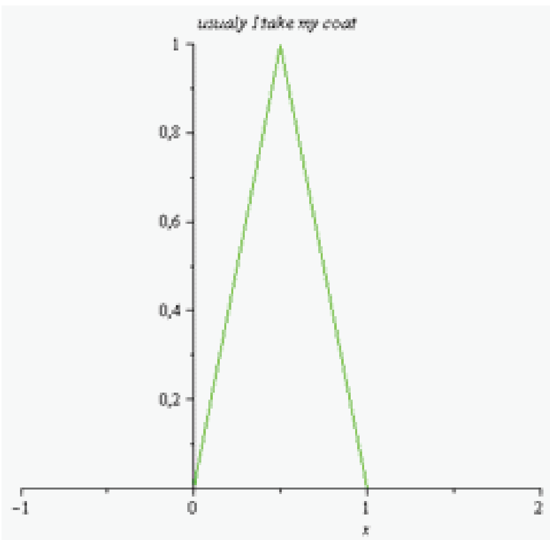

Figure 3.3: The fuzzy word 'usually I take my coat' membership function.

Definition 3.4

The 'product fuzzy logic operator *AND*' transform the fuzzy logic expression [(a_1 is A_1) *AND* (a_2 is A_2)] in the fuzzy statement denoted usually by

[(a_1 is A_1) $\bowtie_{product\text{-}intersection}$, ($a_2$ is A_2)], and represented by the fuzzy subset $C_{product\text{-}intersection}$, which membership function is

$$f_{C_{product-intersection}}\left(a_1,a_2\right)=f_{A_1}\left(a_1\right)\times f_{A_2}\left(a_2\right). \tag{3.4}$$

For the premises $a_1=5^0C$ and $a_2=500\text{m}f_{A_1}\left(a_1\right)=0.6666666667$ and $f_{A_2}\left(a_2\right)=0.2307692308$

Therefore: $f_{C_{minimum-intersection}}\left(a_1,a_2\right)=\text{minimum}[f_{A_1}\left(a_1\right),f_{A_2}\left(a_2\right)]=0.2307692308$

$f_{C_{product-intersection}}\left(a_1,a_2\right)=f_{A_1}\left(a_1\right)\times f_{A_2}\left(a_2\right)=0.1538461538.$

In case of the example 2.1, the *DOF* of the fuzzy logic expression [(a_1 is A_1) *AND* (a_2 is A_2)] using

The 'minimum logic operator *AND'* is equal to 0.2307692308.

And using the 'product logic operator *AND*' the *DOF* of the fuzzy logic expression (a_1 is A_1) *AND* (a_2 is A_2) is equal to 0.1538461538.

This means that the meaning of the fuzzy logic expression (a_1 is A_1) *AND* (a_2 is A_2) is not unique. In other words beside the premises a_1,a_2 the *DOF* of the fuzzy logic

Expression (a_1 is A_1) *AND* (a_2 is A_2) depends also on the meaning of the fuzzy logic operator *AND*.

This dependence shows that the $DOF\ of\ the\ rule\ 'If\left(a_1 is\ A_1\right)AND(a_2 is\ A_2)then\ (b\ is\ B)'$ defined in [2] Definition 3.1 pg.47.as being equal to that of the fuzzy logic expression is not unique.

Beside the premises, a_1,a_2 this depend also on the meaning of the fuzzy logic operator *AND*.

Remark that in case of Boolean logic model, because the algebraic operations 'minimum' and 'product' defined in the set of numbers {0,1}

Coincides, the above underlined differences does not exist.

4. What is the true value of fuzzy logic reasoning in 'human intelligence' variable,using the possibility distribution?

In general using the truth-value (i.e. the degree of fulfillment *(DOF)*) corresponding to the fulfillment of the conditions of a fuzzy logic expression

A = (a_1 *is* A_1) $\bowtie_1$ (a_2 is A_2) $\bowtie_2$ ... $\bowtie_{k-1}$ (a_k *is* A_k) for given premises a_1,a_2, ... a_k according to [2] pg.45-46 the possibility distribution on the resulting set

V = the set where the fuzzy set B is included, can be calculated following the method by Dubois and Prade [3].

Let be $U=the\ set\ where\ the\ fuzzy\ sets\ A_1,A_2,\dots A_k\ are\ included.$

Definition.4.1

The conditional possibility distribution $\Pi_{B\nmid A}$ formula for any given v is

$$\Pi_{B\nmid A}(v)=sup_u\{t(\Pi_{B\nmid A}(v,u),\Pi_A(u))\} \tag{4.1}$$

With t being, a triangular norm defined as in Definition 2.7. pg.11. [2].

Statement.4.1

In this case, using the membership functions f_A and f_B and as possibility distributions on obtain the inequality:

$$f_B(v) \geq sup_u \{t\Pi_{B \nmid A}(v,u), f_A(u))\} \tag{4.2}$$

Statement 4.2.

If the *t norm selected is the minimum then the conditional possibility distribution* $\Pi^{min}_{B \nmid A}$ can be given as.

$$\Pi^{min}_{B \nmid A}(u,v) = 1 \, if \; f_A(u) \leq f_B(v), \text{ and } \Pi^{min}_{B \nmid A}(u,v)(u,v) = f_B(v) \; if \quad f_A(u) > f_B(v) \tag{4.3}$$

Statement 4.3.

If the *t norm selected is the product then the conditional possibility distribution* $\Pi^{prod}_{B \nmid A}$ *can be given as.*

$$\Pi^{prod}_{B \nmid A}(u,v) = 1 \, if \; f_A(u) = 0, \text{ and } \Pi^{prod}_{B \nmid A}(u,v) = \min\{1, \frac{f_B(v)}{f_A(u)}\} \; if \quad f_A(u) > 0 \tag{4.4}$$

In case of the example 3.1. $u = (a_1, a_2)$ *and* $v = b$.

For $u = (5, 500)$ $f_{C_{min-intersection}}(u) = \min[f_{A_1}(a_1), f_{A_2}(a_2)] = 0.2307692308$ and $f_{C_{prod-intersection}}(u) = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.1538461538$.

For $v = 0.25$ $f_B(v) = 0.5$ *and for* $v = 0.9$ $f_B(v) = 0.2$.

If the min-intersection= $\bowtie_{min\text{-}intersection}$ and *the minimum t norm are selected then for*

$u = (5, \; 500)$ *and* $v = 0.25$ *we have* $f_A(u) = 0.2307692308$ and $f_B(v) = 0.5$ *therefore* $f_A(u) \leq f_B(v)$ *and* $\Pi^{min}_{B \nmid A}(u,v) = 1$

If the min-intersection= $\bowtie_{min\text{-}intersection}$ and *the minimum t norm are selected then for*

$u = (5, \; 500)$ *and* $v = 0.9$ *we have* $f_A(u) = 0.2307692308$ and $f_B(v) = 0.2$ *therefore* $f_A(u) > f_B(v)$ *and* $\Pi^{min}_{B \nmid A}(u,v) = 0.2$

If the min-intersection= $\bowtie_{min\text{-}intersection}$ and *the minimum t norm are selected then for*

$u = (5, \; 500)$ *and* $v = 0.25$ *we have* $f_A(u) = 0.2307692308$ and $f_B(v) = 0.5$ *therefore* $\min\left\{1, \frac{f_B(v)}{f_A(u)}\right\}$

$= \min\left\{1, \frac{0.5}{0.2307692308} = 2.166666666\right\} = 1$ *and we have* $\Pi^{prod}_{B \nmid A}(u,v) = 1$

If the min-intersection= $\bowtie_{min\text{-}intersection}$ and *the minimum t norm are selected then for*

$u = (5, \; 500)$ *and* $v = 0.9$ *we have* $f_A(u) = 0.2307692308$ and $f_B(v) = 0.2$ *therefore* $\min\left\{1, \frac{f_B(v)}{f_A(u)}\right\}$

$= \min\left\{1, \frac{0.2}{0.2307692308} = 0.8666666666\right\} = 0.8666666666$ *and we have* $\Pi^{prod}_{B \nmid A}(u,v) = 0.8666666666$

If the prod-intersection= $\bowtie_{prod\text{-}intersection}$ and *the minimum t norm are selected then for*

$u = (5, \; 500)$ *and* $v = 0.25$ *we have* $f_A(u) = 0.1538461538$ and

$f_B(v) = 0.5$ *therefore* $f_A(u) \leq f_B(v)$ *and* $\Pi^{min}_{B \nmid A}(u,v) = 1$

If the prod-intersection= $\bowtie_{prod\text{-}intersection}$ and *the minimum t norm are selected then for*

$u = (5, \; 500)$ *and* $v = 0.9$ *we have* $f_A(u) = 0.1538461538$ and $f_B(v) = 0.2$ *therefore* $f_A(u) \leq f_B(v)$ *and* $\Pi^{min}_{B \nmid A}(u,v) = 1$

If the prod-intersection= $\bowtie_{prod\text{-}intersection}$ and *the minimum t norm are selected then for*

$u=(5{,}500)$ *and* $v=0.25$ *we have* $f_A(u)=0.1538461538$ and

$$f_B(v)=0.5 \; therefore \; \min\left\{1,\frac{f_B(v)}{f_A(u)}\right\}=\min\left\{1,\frac{0.5}{0.1538461538}=3.250000001\right\} and \; \Pi^{prod}_{B\nmid A}(u,v)=1$$

If the prod-intersection= $\bowtie_{prod\text{-}intersection}$ and *the minimum t norm are selected then for*

$$u=(5{,}500) \; and \; v=0.9 \; we \; have \; f_A(u)=0.1538461538 \text{ and } f_B(v)=0.2 \; therefore \; \min\left\{1,\frac{f_B(v)}{f_A(u)}\right\}=\min\left\{1,\frac{0.2}{0.1538461538}=1.300000000\right\} and \; \Pi^{prod}_{B\nmid A}(u,v)=1$$

The computed results above raise several key observations and questions:

-the *DOF* of a fuzzy logic expression $A=(a_{1\text{ is}}A_1)\bowtie(a_{2\text{ is}}A_2)$ for given premises a_1,a_2 is highly dependent of the considered kind of intersection $\bowtie_{min\text{-}intersecion}$, $\bowtie_{prod\text{-}intersecion}$ i.e. min-intersection or prod-intersection. For the same given premises a_1,a_2 the $DOF_{min\text{-}intersecion}$(A) and $DOF_{prod\text{-}intersecion}$*(A)* can be significantly different. In case of example 2.1 for $u=(5{,}500)$ *DOF* {[5 is A_1) $\bowtie_{min\text{-}intersecion}$ (500 is A_2)]} = 0.2307692308 and *DOF* {[5 is A_1) $\bowtie_{prod\text{-}intersecion}$ (500 is A_2)]} = 0.1538461538. In this situation, the question is: which of the two intersections $\bowtie_{min\text{-}intersecion}$, $\bowtie_{prod\text{-}intersecion}$ is appropriate to represent the degree of fulfillment of the rule *if* $A=(a_{1\text{ is}}A_1)\bowtie(a_{2\text{ is}}A_2)$ then (*b* is *B*)? (see [2] pg.47. Definition 3.1.)

- the conditional possibility distribution $\Pi_{B\nmid A}$ of the fuzzy logic rule of reasoning *if* $A=(a_{1\text{ is}}A_1)\bowtie(a_{2\text{ is}}A_2)$ then (*b* is *B*) for given premises a_1,a_2 and given choice of intersection $\bowtie_{min\text{-}intersecion}$, $\bowtie_{prod\text{-}intersecion}$ is highly dependent of the choice of *the t norm*. In case of the example 2.1. for $u=(5\,,500)\,,n_{min\text{-}intersection}\,,v=0.9$ *and minimum t norm, we obtain* $\Pi^{min}_{B\nmid A}(u,v)=0.2$ and for $u=(5\,,500)\,,n_{min\text{-}intersection},v=0.9$ *, and prod t norm* $\Pi^{prod}_{B\nmid A}(u,v)=0.8666666666$.

In this situation, the question is: which of these values is appropriate to represent the degree of fulfillment of the rule *if* $A=(a_{1\text{ is}}A_1)\bowtie(a_{2\text{ is}}A_2)$ then (*b* is *B*) ? see [2] pg.47. Definition 3.1.

-this dependence shows that the '$DOF \; of \; the \; rule \; If \left(a_1 is \, A_1\right) AND (a_2 is \, A_2) then \; (b \, is \, B)$' defined as being equal to the conditional possibility distribution $\Pi_{B\nmid A}$ is questionable.

5. What means minimum, maximum and additive combination in the framework of the 'human intelligence' linguistic variable?

The analysis presented in the previous sections reveal that in general several rules can be derived for the same situation expressed as a vector of premises. That is because for the same premises $a_1,a_1,\ldots,a_k$ and the same fuzzy sets $A_1,A_1,\ldots,A_k$ a rule has different logical expression $A^i=(a_{1\text{ is}}A_1)\bowtie^i_1(a_{2\text{ is}}A_2)\bowtie^i_2\ldots\bowtie^i_{k-1}(a_{k\text{ is}}A_k)$ due to the logical operators $\bowtie^i_1\bowtie^i_2,\ldots\bowtie^i_{k-1}$. The different logical.

Expressions A^i has different $DOF(A^i)$ and different consequences B_i.The rules

If $(a_{1\text{ is}}A_1)\bowtie^i_1(a_{2\text{ is}}A_2)\bowtie^i_2\ldots\bowtie^i_{k-1}(a_{k\text{ is}}A_k)$ then (b_i is B_i)

Has different true values = $DOF\left(IfA^i then \; (b \, is B_i)\right)$.

Therefore an overall response has to be derived for the set of rules. This overall response has to be a combination of a few individual rule responses that takes into consideration. The individual *F – s.*

There are several possibilities to combine rule responses. [4] the most common ones are the Minimum, maximum and additive combination methods.

Definition 5.1

The minimum combination of responses $(DOF(A^i),B_i)$ is the fuzzy set *B* with the membership function

$$f_B(x)=minimum[DOF(A^i)\times f_{B_i}(x))]_{(DOFA^i)>0} \tag{5.1}$$

The minimum combination tries to find a combined rule response, which at least to a certain level is in agreement with all applicable rules. Thus the philosophy for minimum combination fuzzy responses considers only those elements as possible consequences that have a positive $DOF\left(A^i\right)>0$.

Definition 5.2

The cresting minimum combination of responses $(DOF\left(A^i\right), B_i)$ is the fuzzy set B with the membership function

$$f_B(x)=minimum[minimum\left(DOF\left(A^i\right), f_{B_i}(x)\right)]_{(DOFA^i)>0} \quad (5.2)$$

Example 2.2.([2] pg.60.)Consider the following rule responses:

Rule 1 has the $DOF\left(A^1\right)=0.4\, and\ the\ B_1=(0,2,4)\, triangular\ fuzzy\ set.$

Rule 2 has the $DOF\left(A^2\right)=0.4\, and\ the\ B_2=(3,4,5)\, triangular\ fuzzy\ set.$

No other rule apply $(DOF\left(A^i\right)=0\, if\ i>2)$

As the supports of B_1 and B_2 have the intersection [3,4] *the minimum is zero outside this interval.* The minimum combination is thus obtained by taking the minimum of $\min\left(0.4\times\frac{4-x}{2}, 0.5\times(x-3)\right)$ $on\ [3,4]$

As the equation $0.4\times\frac{4-x}{2}=0.5\times(x-3)$ $has\ the\ solution\ \frac{23}{7}$ the minimum is

$f_B(x)=0.5\times(x-3)\, if\ 3<x\le\frac{23}{7}$ and $f_B(x)=0.4\times\frac{4-x}{2}$ $if\ \frac{23}{7}<x<4$.

The membership function of the consequence B of the minimum combination is represented on the next (Figure 5.1) with color red.

The cresting minimum combination is obtained by taking the minimum of the two membership functions

f_{B_1}, f_{B_2} $and\ the\ two\ fullfillments\ grade\ DOF\left(A^1\right), DOF\left(A^2\right).$

The equation $\frac{4-x}{2}=x-3$ $has\ the\ solution\ \frac{10}{3}$., the minimum of the two functions and the two constants are thus found to be: $f_B(x)=(x-3)$ if $3<x\le\frac{10}{3}$ and $f_B(x)=\frac{4-x}{2}$ if $\frac{10}{3}<x<4$. The membership function of the consequence B of the crisp minimum combination is represented on the next (Figure 5.2) with color red.

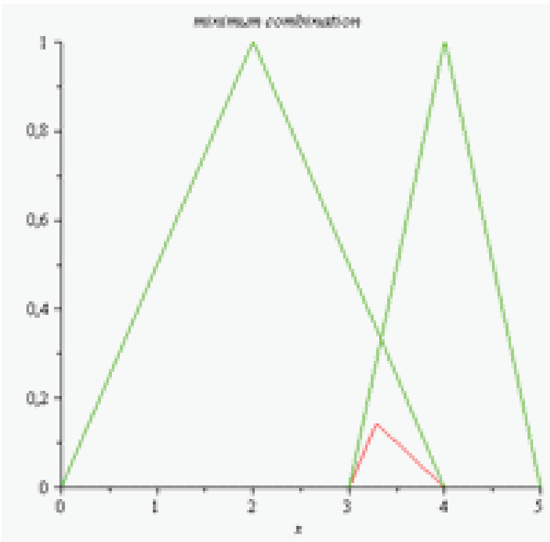

Figure 5.1: The minimum combination. Depicted in red.

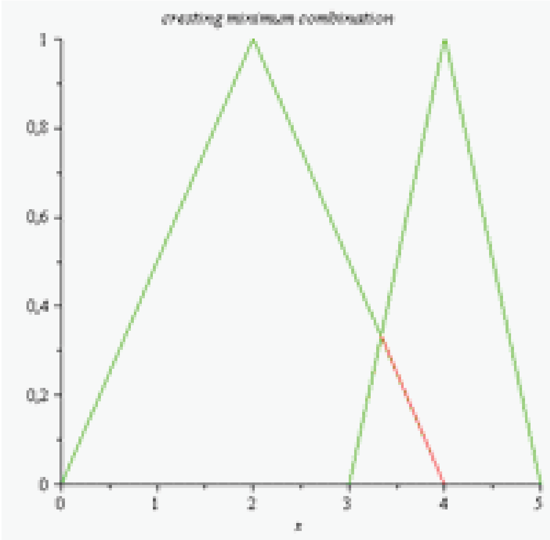

Figure 5.2: The cresting minimum combination. Depicted in red.

Remark. A disadvantage of the minimum combination methods such as ones defined by Eqs. (5.1), (5.2) is that any disagreement impairs the usage of the rule system. In other words, if the rules are not carefully constructed then it may happen that the combination of responses leads to $f_B(x)=0$ *for some* $a_1,a_2,\ldots,a_k$ *premeises.* Therefore , use of minimum combinations requires more care than that of other combination methods. To avoid the aforementioned problem of possible disagreement, on can define a response combination method for which an outcome *b* in the set of possible responses becomes possible (having a positive membership value) if there is at least one rule with positive $(DOF(A^i)$ for which that positive outcome was a possible response. In this case, only an agreement on the impossible responses is required (agreement preservation of impossible responses). In other words if an outcome has zero membership function for all rule responses for which the rule has positive $(DOF(A^i)$ then the outcome has also a zero membership in the combined response. These requirements may be fulfilled using the maximum combination method [5].

Definition 5.3

The maximum combination of responses $(DOF(A^i),B_i)$ is the fuzzy set *B* with the membership function

$$f_B(x)=\max[DOF(A^i)\times f_{B_i}(x))]_{i=1,2\ldots} \tag{5.3}$$

On can also crest the membership functions instead of multiplying them by the fulfillment grade.

Definition 5.4

The cresting maximum combination of responses $(DOF(A^i),B_i)$ is the fuzzy set *B* with the membership function

$$f_B(x)=\max[\min\left(DOF\left(A^i\right),f_{B_i}(x)\right)]_{i=1,2,\ldots} \tag{5.4}$$

Remark. The maximum combination method tolerate disagreements, but they do not emphasize eventual agreements. More precisely the event of two rules giving the same result does not induce an increase of the membership function of the response, thus it has no effect on the credibility of the result. If for example the rules represent expert opinions this insensitivity to the proportion of experts agreeing, is not a desirably property [6]. Furthermore, the maximum combinations overemphasize rules with very vague responses. For example *If A^1 is anything and A^2 is anything then B can anything* would dominate all the other rules because it would exhibit both high *DOF* - *s* and high membership function $f_{B_i}(x)$

For the same data considered in Example 2.2., the maximum combination is obtained by taking the maximum of the two membership functions, which yields the membership function:

$$f_B(x)=0.4\times\frac{x}{2} \text{ if } 0<x\le 2\ ,\ f_B(x)=0.4\times\frac{4-x}{2}\ \text{ if } 2<x\le\frac{23}{7},$$
$$f_B(x)=0.5\times(x-3) \text{ if } \frac{23}{7}<x\le 4\ ,f_B(x)=0.5\times(5-x)\ \text{ if }\ 4<x\le 5$$

The membership function of the consequence *B* of the maximum combination is represented on the next (Figure 5.3) with red color.

The cresting maximum combination require more algebraic effort. First, the minimum of the fulfillment grades and then the membership functions have to be calculated.

The membership function of the consequence *B* of the cresting maximum combination is

$$f_B(x) = \frac{x}{2} if\ 0 < x \le 0.8,\ f_B(x) = 0 if\ 0.8 < x \le 3.2,$$

$$f_B(x) = \frac{4-x}{2} if\ 3.2 < x \le \frac{10}{3},$$

$$f_B(x) = x - 3 if\ \frac{10}{3} < x \le 3.5,$$

$$f_B(x) = 0.5 if\ \frac{10}{3} < x \le 4.5,$$

$$f_B(x) = 5 - x if\ 4.5 < x \le 5$$

and is represented on the next (Figure 5.4) with color blue.

A possible compromise between the minimum and maximum combination methods, one may select one of the following types of additive combinations. Namely the weighted sum, the normed weighted sum, the cresting weighted sum, the cresting normed weighted sum combinations. Additive combinations have been proposed in [6]

Definition 5.5

The weighted sum of combination of responses $(DOF(A^i), B_i)$ is the fuzzy set *B* with the membership function

$$f_B(x) = \frac{\sum_{i=1}^{I} DOF(A^i) \times f_{B_i}(x)}{max_u[\sum_{i=1}^{I} DOF(A^i) \times f_{B_i}(u)]} \tag{5.5}$$

The division by the maximum of the sum is required to ensure that the resulting membership function is not greater than 1.

In general on can state that a rule is better if its consequence is more specific. A rule with the response 'anything 'has no value at all. To consider this specificity when consequences are very different in vagueness, another combination method can be defined, namely, the normed weighted sum combination.

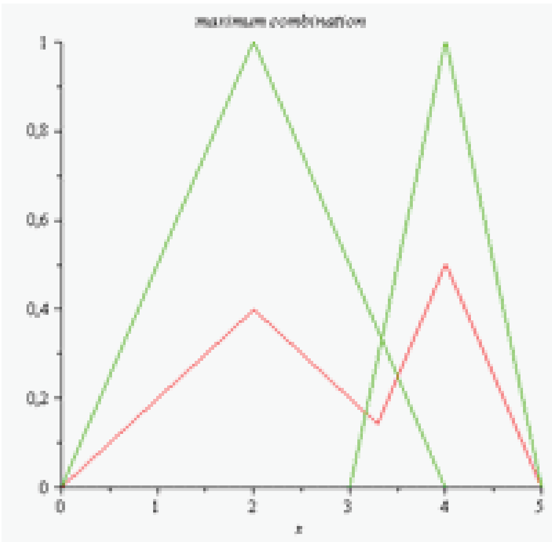

Figure 5.3: The maximum combination. Depicted in red.

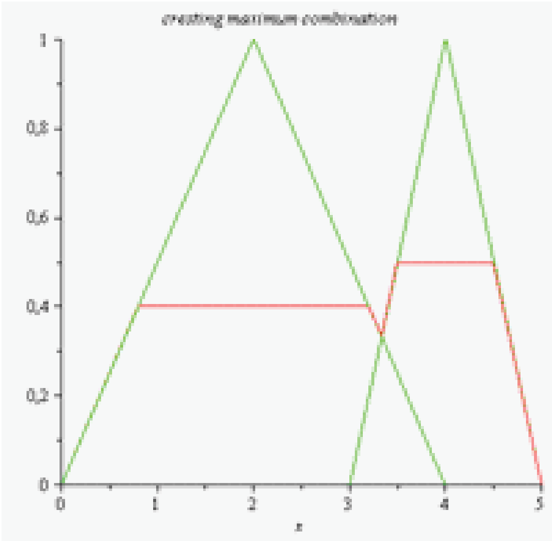

Figure 5.4: The cresting maximum combination. depicted in red.

Definition 5.6

The normed weighted sum of combination of responses $(DOF\left(A^i\right), B_i)$ is the fuzzy set B with the membership function

$$f_B(x) = \frac{\sum_{i=1}^{I} DOF\left(A^i\right) \times \beta_i \times f_{B_i}(x)}{max_u[\sum_{i=1}^{I} DOF\left(A^i\right) \times \beta_i \times f_{B_i}(u)]} \quad (5.6)$$

$$where \quad \beta_i = \frac{1}{\int_{-\infty}^{\infty} f_{B_i}(x)dx}$$

In this case each consequence B_i is assigned a unit weight but rules have different weights. Rules with crisper answers carry greater weight than rules very fuzzy (Uncertain) answer.

The cresting version of the above additive combination methods can be also defined:

Definition 5.7

The cresting weighted sum of combination of responses $(DOF\left(A^i\right), B_i)$ is the fuzzy set B with the membership function:

$$f_B(x) = \frac{\sum_{i=1}^{I} \min[DOF\left(A^i\right), f_{B_i}(x)]}{max_u[\sum_{i=1}^{I} \min[DOF\left(A^i\right), f_{B_i}(u)]} \quad (5.7)$$

Here again the division by the maximum of the sum is required to ensure that the resulting membership function is not greater than 1.

The normed version of this combination method can be defined in an analogous manner as:

Definition 5.8

The cresting normed weighted sum combination of responses $(DOF\left(A^i\right), B_i)$ is the fuzzy set B with the membership function:

$$f_B(x) = \frac{\sum_{i=1}^{I} \beta_i \times \min[DOF\left(A^i\right), f_{B_i}(x)]}{max_u[\sum_{i=1}^{I} \beta_i \times \min[DOF\left(A^i\right), f_{B_i}(u)]} \quad (5.8)$$

$$where \quad \beta_i = \frac{1}{\int_{-\infty}^{\infty} f_{B_i}(x)dx}$$

Remark. Additive combinations of rules take the agreement of responses into account since by virtue of adding the membership functions, the membership of elements with such agreements increases. On the other hand no responses that were impossible for each rule with positive *DOF* may become possible by additive combination method. The support of the result of the additive and the maximum combination method is the same.

The combination methods fulfill a number of rational requirements. The most important ones are listed here:

-Idempotency: if a response is combined with itself then the combined response should not be altered:

$$C((DOF\left(A^{i}\right),B_{i}),(DOF\left(A^{i}\right),B_{i})=C(\left(DOF\left(A^{i}\right),B_{i}\right))$$

Note that this property does not mean that the response should be equal to the unique response. Combination may alter single responses, for example, by cresting.

$$-\text{Associativity}: C((DOF\left(A^{1}\right),B_{1}),C(\left(DOF\left(A^{2}\right),B_{2}\right),\left(DOF\left(A^{3}\right),B_{3}\right)))$$
$$=C(C((DOF\left(A^{1}\right),B_{1}),\left(DOF\left(A^{2}\right),B_{2}\right)),\left(DOF\left(A^{3}\right),B_{3}\right)).$$

$$-\text{Symmetry}: C((DOF\left(A^{1}\right),B_{1}),(DOF\left(A^{2}\right),B_{2})=C((DOF\left(A^{2}\right),B_{2}),(DOF\left(A^{1}\right),B_{1})$$

The associativity and the symmetry ensure that the combination of the rule responses can be calculated in any order without altering the final result.

For the same data considered in Example 2.2.the weighted sum combination is obtained by taking first the sum of the two membership functions, which is the maximum of the two Membership functions:

$$f(x)=0.4\times\frac{x}{2} if\ 0<x\le 2\ ,\ f(x)=0.4\times\frac{4-x}{2}\ if\ if\ 2<x\le 3\ ,$$
$$f(x)=0.5\times(x-3)+0.4\times\frac{4-x}{2} if\ \ 3<x\le 4\ \ ,f(x)=0.5\times(5-x)\ if\ \ 4<x\le 5$$

As the maximum of $f(x)$ is 0.5 the membership function of B is $f_B(x) = 2 \times f(x)$.

The membership function of the consequence B of the weighted sum combination is represented on the next (Figure 5.5).

For the same data considered in Example 2.2.the normed weighted sum combination is obtained by computing

$$first\ \beta_1=\frac{1}{\int_{-\infty}^{\infty} f_{B_1}(x)dx}=0.5\, and\ \ \beta_2=\frac{1}{\int_{-\infty}^{\infty} f_{B_2}(x)dx}=1\ .$$

Therefore the normed

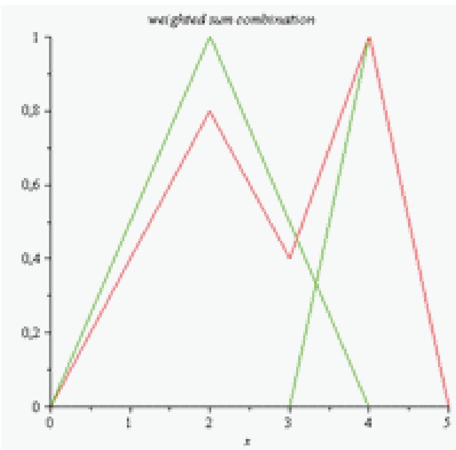

Figure 5.5: The weighted sum combination. Depicted in red.

Weighted sum combination is obtained by taking the weighted sum function first:

$$f(x) = 0.2 \times \frac{x}{2} \text{ if } 0 < x \le 2, \; f(x) = 0.2 \times \frac{4-x}{2} \text{ if if } 2 < x \le 3,$$
$$f(x) = 0.5 \times (x-3) + 0.2 \times \frac{4-x}{2} \text{ if } \; 3 < x \le 4, \; f(x) = 0.5 \times (5-x) \text{ if } \; 4 < x \le 5$$

As the maximum of $f(x)$ is 0.5 the membership function of B is obtained as $f_B(x) = 2 \times f(x)$.

The membership function of the consequence B of the normed weighted sum combination is represented on the next (Figure 5.6).

Remark. The difference between the weighted sum combination and the normed weighted sum

Combination lies in the treatment of the fuzziness of the consequences. The weighted sum combination does not account for the different uncertainties inherent to the consequence elements. In contrast, the normed weighted sum combination places more weight on the crisper consequence.

For the same data considered in Example 2.2. The cresting weighted sum combination is obtained by first taking the minimum of the

DOF - s grade and the membership functions and then summing the two functions:

$$f(x) = \frac{x}{2} \text{ if } 0 < x \le 0.8, \; f(x) = 0.4 \text{ if } 0.8 < x \le 3, \; f(x) = x - 2.6 \text{ if } 3 < x \le 3.2,$$
$$f(x) = \frac{4-x}{2} + x - 3 \text{ if } 3.2 < x \le 3.5,, \; f(x) = \frac{4-x}{2} + 0.5 \text{ if } 3.5 < x \le 4,$$
$$f(x) = 0.5 \text{ if } 4 < x \le 4.5, \; f(x) = 5 - x \text{ if } 4.5 < x \le 5,$$

As the maximum of $f(x)$ is 0.75 the membership function of B in case of the cresting weighted sum combination is $f_B(x) = \frac{f(x)}{0.75}$. The membership function of the consequence B of the cresting weighted sum combination is represented on the next (Figure 5.7).

For the same data considered in Example 2.2.the cresting normed weighted sum combination is obtained by computing $\textit{first } \beta_1 = \frac{1}{\int_{-\infty}^{\infty} f_{B_1}(x)dx} = 0.5 \textit{ and } \; \beta_2 = \frac{1}{\int_{-\infty}^{\infty} f_{B_2}(x)dx} = 1.$ Therefor the cresting normed weighted sum combination is obtained by taking the weighted sum function $f(x)$ first

$$f(x) = 0.5 \times \frac{x}{2} \text{ if } 0 < x \le 0.8, \; f(x) = 0.2 \text{ if } 0.8 < x \le 3, \; f(x) = x - 2.8 \text{ if } 3 < x \le 3.2,$$
$$f(x) = 0.5 \times \frac{4-x}{2} + x - 3 \text{ if } 3.2 < x \le 3.5, \; f(x) = 0.5 \times \frac{4-x}{2} + 0.5 \text{ if } 3.5 < x \le 4,$$
$$f(x) = 0.5 \text{ if } 4 < x \le 4.5, \; f(x) = 5 - x \text{ if } 4.5 < x \le 5.$$

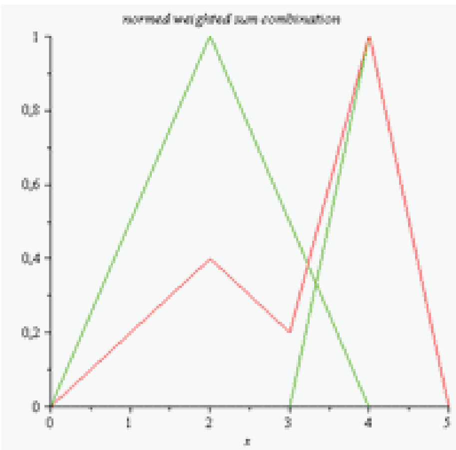

Figure 5.6: The normed weighted sum combination, depicted in red.

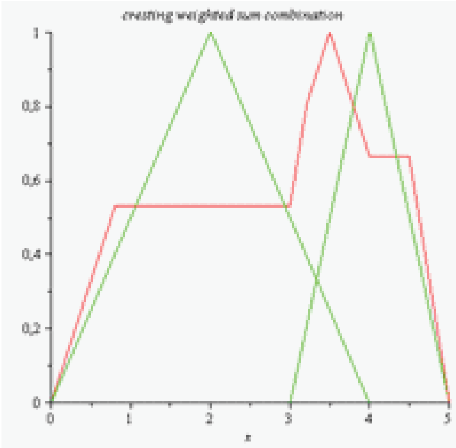

Figure 5.7: The cresting weighted sum combination. Depicted in red.

As the maximum of $f(x)$ is 0.7, the membership function of B in case of the cresting normed weighted sum combination is $f_B(x) = \frac{f(x)}{0.7}$.

The membership function of the consequence B of the cresting normed weighted sum combination is represented on the next (Figure 5.8).

The results presented in this section show a wide variety of different overall responses that can be obtained, in the framework of 'human intelligence' linguistic variable, starting from a set of two simple reasoning, that refer the same premises. Figures 5.1-5.8 permit a rough comparison of the fuzzy set of an overall responses with the fuzzy sets of the two starting responses.

6. What means the defuzzyfication of a fuzzy consequence?

It is often necessary to replace the fuzzy consequence B, obtained as a combination of individual rule consequences with a single crisp consequence B^*. For example a point prediction can be required for forecasting, decision making control**.**

Definition 6.1

The procedure that transforms a fuzzy consequence B into a crisp consequence B^* is called defuzzification [2] pg.70.

Definition 6.2

The defuzzification by maximum of the fuzzy consequence B selects the element B^* with the maximum membership value as the representative element of the consequence fuzzy set B [2] pg.71.

$$f_B(b^*) = max_x\left[f_B(x)\right] \tag{6.1}$$

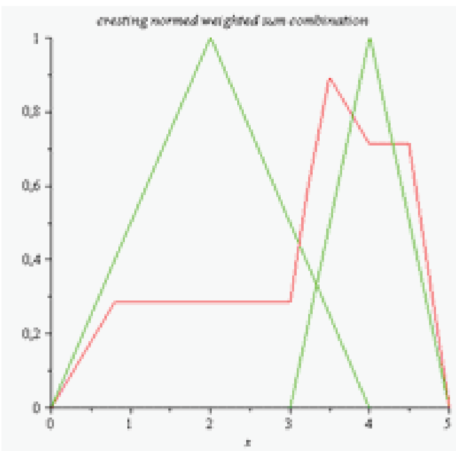

Figure 5.8: The cresting normed weighted sum combination. Depicted in red.

Note that B^* defined by (6.1) is not necessarily unique. It can happen that there exists an infinite number of elements x having the maximal value as membership as in Figure 3.1.

In continuous cases, rules yield consequences that are fuzzy sets defined over the real line that are fuzzy set defined on the real line with continuous membership functions the defuzzification can be done also by using another location parameter, for example, the fuzzy mean $M(B)$ of B [2].

$$b^* = M(B) = \frac{\int_{-\infty}^{\infty} t \times f_B(t)dt}{\int_{-\infty}^{\infty} f_B(t)dt} \tag{6.2}$$

The median defuzzification of the fuzzy consequence B is its fuzzy median $m(B)$

$$b^* = m(B) \, defined \, by \int_{-\infty}^{m(B)} f_B(t)dt = \int_{m(b)}^{\infty} f_B(t)dt \tag{6.3}$$

The defuzzified responses for example 2.2.are as follows:

Combination method	Defuzzification method		
	Maximum	Mean	Median
Minimum	3.287	3.429	3.402
Cresting minimum	3.333	3.444	3.428
Maximum	4.000	2.731	2.638
Cresting maximum	[3.5, 4.5]	2.676	2.729
Weighted sum	4.000	2.769	2.775
Normed weighted sum	4.000	3.111	3.309
Cresting weighted sum	3.500	2.739	2.743
Cresting normed weighted sum	3.500	3.089	3.279

7. The effect of incorporation of some fuzzy logic expression from the 'human intelligence' linguistic variable in the premises of a fuzzy logic reasoning

In this section we illustrate the effect of the incorporation of one of the statements, (a_3 is $A_{intelligent}$), (a_3 is $A_{very\ intelligent}$), (a_3 is $A_{more\ or\ less\ intelligent}$), ($a_3$ is $A_{indeed\ intelligent}$), in the premises of the fuzzy logic reasoning 'If it is cold and I have a long way to walk, then I usually take my coat' Example 3.1.pg.45.[2]

Remember that he fuzzy set A_1 represents the temperature. 'Cold' might be characterized with a membership 1 for -10$C \le T \le$ 0C, or for $T \ge 15C$ *and* $T \le -20$ *and linear between* this is the trapezoidal fuzzy set (-20, -10, 0, 15) which membership function is:

$$f_{A_1}(x) = 0 \; for \; x < -20; \; f_{A_1}(a_1) = \frac{a_1 + 20}{10} \; for -20 \le a_1 < -10; \; f_{A_1}(a_1) = 1$$
$$for -10 \le a_1 \le 0; \; f_{A_1}(x) = \frac{15 - a_1}{15} \; for \, 0 < a_1 \le 15; \; f_{A_1}(a_1) = 0 \; for \, 15 < a_1;$$

The fuzzy expression 'long walk' A_2 can also be characterized by the triangular fuzzy set $A_{2\,=}$ (200, 1500, 4000) meters which membership function is:

$$f_{A_2}(a_2) = 0 \; for \; a_2 < 200 \; ; \; f_{A_2}(a_2) = \frac{a_2 - 200}{1300} \; for \; 200 \le a_2 < 1500 \; ; \; f_{A_2}(a_2) = \frac{4000 - a_2}{2500}$$
$$for \; 1500 \le a_2 < 4000 \; ; \; f_{A_2}(a_2) = 0 \; for \; 4000 \le a_2$$

The fuzzy set B represents the word 'usually' that might be characterized by the triangular fuzzy set (0, 0.5, 1) which membership function is:

$$f_B(b) = 0 \; for \, b < 0 \; ; \; f_B(b) = 2 \times b \; for \, 0 \le b \le 0.5 \; ;$$
$$f_B(b) = -2 \times b + 2 \; for \; 0.5 \le b \le 1 \; ; \; f_B(b) = 0 \; for \, b > 1$$

In this example the fuzzy logic expression is, (a_1 *is* A_1) *AND* (a_2 *is* A_2) and the fuzzy logic reasoning is, *If* (a_1 *is* A_1)*AND*(a_2 *is* A_2) *then* (b *is* B).

Consider now, the fuzzy statement (a_3 *is* A_3), where the fuzzy set A_3 is the triangular fuzzy set which membership function is

$$f_{A_3}(a_3)=0 \ \ for \ a_3<40 \ ; f_{A_3}(a_3)=\frac{1}{60}\times(a_3-40) \ for \ 40\le a_3<100;$$

$$f_{A_3}(a_3)=-\frac{1}{60}\times(a_3-160) \ for \ 100\le a_3<160 \ ; f_{A_3}(a_3)=0 \ \ for \ 160\le a_3$$

This fuzzy subset correspond to what we call general intelligence evaluated with IQ index and we denote by $A_{intelligent}$ i.e. *(*$A_3 = A_{intelligent}$*)*.

-Case when $A_3 = A_{intelligent}$ [8].

The new fuzzy logic expression is

(a_1 *is* A_1) *AND* (a_2 *is* A_2) *AND* *(*a_3 *is* $A_{intelligent}$*)* And

The new fuzzy logic reasoning is

IF (a_1 *is* A_1) *AND* (a_2 *is* A_2) *AND (*a_3 *is* $A_{intelligent}$*) then (b is B)*.

Retain that in the new fuzzy logic expression and in the new fuzzy logic reasoning a new statement appear. That is *(*a_3 *is* $A_{intelligent}$*)* and concern the general human intelligence evaluated by the IQ index. The consequence is the same as in the previous logic reasoning.

Computing the *DOF - s* of the new fuzzy logic expression and comparing them with those obtained in section 3 the effect of the inclusion in the premises of the statement *(*a_3 *is* $A_{intelligent}$*)* can be seen**.**

In the following we give a more detailed analysis concerning this effect

Using the min fuzzy logic operator *AND* the membership function of the new fuzzy logic expression (a_1 *is* A_1) *AND* (a_2 *is* A_2) *AND (*a_3 *is* A_3*)* is

$$f_{min}(a_1,a_2,a_3)=\min\left\{f_{A_1}(a_1),f_{A_2}(a_2),f_{A_3}(a_3)\right\}=\min\{\min[f_{A_1}(a_1),f_{A_2}(a_2)],f_{A_3}(a_3)\}.$$

Using the prod fuzzy logic operator *AND* the membership function of the new fuzzy logic expression (a_1 *is* A_1) *AND* (a_2 *is* A_2) *AND (*a_3 *is* A_3*) is*

$$f_{prod}(a_1,a_2,a_3)=f_{A_1}(a_1)\times f_{A_2}(a_2)\times f_{A_3}(a_3)=[f_{A_1}(a_1)\times f_{A_2}(a_2)]\times f_{A_3}(a_3).$$

For $a_1=5^0C$ *and* $a_2=500m$ $\min\,[f_{A_1}(a_1),f_{A_2}(a_2)]=0.2307692308.$

Therefore $$f_{min}(a_1,a_2,a_3)=\min\left\{0.2307692308,f_{A_3}(a_3)\right\}$$
$$\le\min\{0.2307692308,1\}=0.2307692308$$

For $a_1=5^0C$ *and* $a_2=500m$ $f_{A_1}(a_1)\times f_{A_2}(a_2)=0.1538461538.$ Therefore

$$f_{prod}(a_1,a_2,a_3)=0.1538461538\times f_{A_3}(a_3)\le 0.1538461538\times 1=0.1538461538.$$

-In case of the use of the min fuzzy logic operator *AND* [8] for

$$a_3<40(\text{very low IQ index})\text{ and }160\le a_3(\text{very high IQ index})\ f_{min}(a_1,a_2,a_3)=0\ .$$

In general

$$f_{min}(a_1,a_2,a_3)<f_{min}(a_1,a_2)\text{ and only for } a_3=100,\ f_{min}(a_1,a_2,100)=0.2307692308=f_{min}(a_1,a_2).$$

-In case of using the 'minimum logic operator *AND*' we have:

$DOF_{min}[(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})] < DOF_{min}[(a_1 is\ A_1)AND(a_2 is\ A_2)]\ and\ only\ for$

$a_3 = 100\ DOF_{min}[(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})] = DOF_{min}[(a_1 is\ A_1)AND(a_2 is\ A_2)] = 0.2307692308.$

On the other hand according to [2] Definition 3.1 pg.47, the *DOF* of the inference using the 'minimum logic operator *AND*' is equal with the *DOF* of the fuzzy logic expression i.e.

$DOF_{min}\left[If\ (a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})then(b\ is\ B)\right]$ is equal to $DOF_{min}[(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})]$.

Hence

$$DOF_{min}\left[If\ (a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})then(b\ is\ B)\right] <$$
$$DOF_{min}\left[If\ (a_1 is\ A_1)AND(a_2 is\ A_2)then(b\ is\ B)\right]\ for\ a_3 \neq 100$$

-In case of the use of the prod fuzzy logic operator *AND* [8] for

$a_3 < 40(very\ low\ IQ\ index)\ and\ 160 \le a_3(very\ high\ IQ\ index)\ f_{prod}(a_1,a_2,a_3) = 0$.

In general

$f_{prod}(a_1,a_2,a_3) < f_{prod}(a_1,a_2)\ and\ only\ for\ a_3 = 100,\ f_{prod}(a_1,a_2,a_3) = 0.1538461538 = f_{prod}(a_1,a_2).$

Moreover

$$DOF_{prod}[(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})] < DOF_{prod}[(a_1 is\ A_1)AND(a_2 is\ A_2)]$$
$$and\ only\ for\ a_3 = 100\ ,DOF_{prod}[(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})] = DOF_{prod}[(a_1 is\ A_1)AND(a_2 is\ A_2)].$$

On the other hand according to [2] Definition 3.1 pg.47, the *DOF* of the inference using the 'prod logic operator *AND*' is equal to the *DOF* of the fuzzy logic expression i.e. $DOF_{prod}\left[If\ (a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})then(b\ is\ B)\right]$ is equal to

$DOF_{prod}[(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})]$.

Hence

$$DOF_{prod}\left[If\ (a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{intelligent})then(b\ is\ B)\right]$$
$$< DOF_{prod}\left[If\ (a_1 is\ A_1)AND(a_2 is\ A_2)then(b\ is\ B)\right]\ for\ a_3 \neq 100$$

- Case, when the fuzzy set A_3 is equal to $A_{very\ intelligent}$. [8]

The membership function is $f_{A_{very\ intelligent}}(a_3) = 0\ for\ a_3 < 40\ ;\ f_{A_{very\ intelligent}}(a_3) = [\frac{1}{60} \times (a_3 - 40)]^2\ for\ 40 \le a_3 < 100;$ $f_{A_{very\ intelligent}}(a_3) = [-\frac{1}{60} \times (a_3 - 160)]^2\ for\ 100 \le a_3 < 160\ ;\ f_{A_{very\ intelligent}}(a_3) = 0\ for\ 160 \le a_3$, and correspond to what we call very intelligent evaluated with IQ index see [8]. Retainthat

$f_{A_{very\ intelligent}}(a_3) \le f_{A_{intelligent}}(a_3)\ for\ any\ a_3$. The new fuzzy logic expression is

$(a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{very\ intelligent})$ and the new fuzzy logic reasoning is

$If\ (a_1 is\ A_1)AND(a_2 is\ A_2)AND\ (a_3 is\ A_{very\ intelligent})\ then(b\ is\ B).$

-In case of the use of the min fuzzy logic operator *AND* [8] for

$a_3 < 40(very\, low\, IQ\, index)\, and\, 160 \le a_3(very\, high\, IQ\, index)\; f_{\min A_{very\, intelligent}}(a_1,a_2,a_3) = 0.$

In general $f_{\min A_{very\, intelligent}}(a_1,a_2,a_3) < f_{\min A_{intelligent}}(a_1,a_2,a_3)\, and\, only\, for\, a_3 = 100,$
$f_{\min A_{very\, intelligent}}(a_1,a_2,a_3) = f_{\min A_{intelligent}}(a_1,a_2,a_3) = 0.2307692308.$

Moreover, $DOF_{min}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})]$
$< DOF_{min}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{intelligent})]\, and\, only\, for\, a_3 =$

$100, DOF_{min}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})] =$
$DOF_{min}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{intelligent})] = 0.2307692308.$

On the other hand according to [2] Definition 3.1 pg.47, the *DOF* of the inference using the 'min logic operator *AND*' is equal to the *DOF* of the logic expression i.e.

$DOF_{min}\left[If\,(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})\, then (b\, is\, B)\right]$
$= DOF_{min}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})].$ Hence

$DOF_{min}\left[If\,(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})\, then (b\, is\, B)\right] <$
$DOF_{min}\left[If\,(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{intelligent})\, then (b\, is\, B)\right]\, for\, a_3 \ne 100$

-In case of the use of the prod fuzzy logic operator *AND* [8] for

$a_3 < 40(very\, low\, IQ\, index)\, and\, 160 \le a_3(very\, high\, IQ\, index)\; f_{prodA very\, intelligent}(a_1,a_2,a_3) = 0.$

In general $f_{prodA very\, intelligent}(a_1,a_2,a_3) < f_{prod\, A intelligent}(a_1,a_2,a_3)\, and\, only\, for\, a_3 = 100,$
$f_{prodA very\, intelligent}(a_1,a_2,a_3) = 0.1538461538 = f_{prod\, A intelligent}(a_1,a_2,a_3).$

Moreover, $DOF_{prod}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{veryintelligent})]$
$< DOF_{prod}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{intelligent})]\, and\, only\, for\, a_3 = 100,$
$DOF_{prod}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intellignt})] =$
$DOF_{prod}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{intelligent})].$

On the other hand according to [2] Definition 3.1 pg.47, the *DOF* of the inference using the 'prod logic operator *AND*' is equal to the *DOF* of the fuzzy logic expression i.e.

$DOF_{prod}\left[If\,(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})\, then (b\, is\, B)\right]$ is equal to

$DOF_{prod}[(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})].$ Hence

$DOF_{prod}\left[If\,(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{very\, intelligent})\, then (b\, is\, B)\right] <$
$DOF_{prod}\left[If\,(a_1 is A_1) AND (a_2 is A_2) AND (a_3 is A_{intelligent})\, then (b\, is\, B)\right]\, for\, a_3 \ne 100$

-Case, when the fuzzy set A_3 is equal to $A_{more\ or\ less\ intelligent}$. [8]

The membership function is

$$f_{A_{more\,or\,less\,intelligent}}(a_3) = 0 \quad for \ a_3 < 40$$

$$f_{A_{more\,or\,less\,intelligent}}(a_3) = \sqrt{\frac{1}{60}\times(a_3 - 40)} \quad for\ 40 \le a_3 < 100,$$

$$f_{A_{more\,or\,less\,intelligent}}(a_3) = \sqrt{-\frac{1}{60}\times(a_3 - 160)} \quad for\ 100 \le a_3 < 160$$

$$f_{A_{more\,or\,less\,intelligent}}(a_3) = 0 \quad for\ 160 \le a_3$$

And correspond to what we call more or less intelligent evaluated with IQ index see [8].

Remark that $f_{A_{very\,intelligent}}(a_3) \le f_{A_{intelligent}}(a_3) \le f_{A_{more\,or\,less\,intelligent}}(a_3)$ $for\ any\ a_3$.

The new fuzzy logic expression is

$$(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_{more\,or\,less\,intelligent})$$

And the new fuzzy logic reasoning is

$$If\ (a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_{more\,or\,less\,intelligent})\ then\ (b\ is\ B).$$

-In case of the use of the min. fuzzy logic operator *AND* [8], for

$$a_3 < 40\ (very\ low\ IQ\ index)\ and\ 160 \le a_3\ (very\ high\ IQ\ index)$$

$$f_{\min A_{more\,or\,less\,intelligent}}(a_1, a_2, a_3) = 0.$$

In general,

$$f_{\min A_{more\,or\,less\,intelligent}}(a_1, a_2, a_3) \ge f_{\min A_{intelligent}}(a_1, a_2, a_3) \ge$$
$$f_{\min A very\,intelligent}(a_1, a_2, a_3)\ and\ only\ for\ a_3 = 100,$$
$$f_{\min A more\,or\,less\,intelligent}(a_1, a_2, a_3) = 0.2307692308 =$$
$$f_{\min A_{intelligent}}(a_1, a_2, a_3) = f_{\min A very\,intelligent}(a_1, a_2, a_3).$$

Moreover,

$$DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_{more\,or\,less\,intelligent})] >$$
$$DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_{intelligent})]\ and\ only\ for$$
$$a_3 = 100,\ the\ next\ equalities\ hold:\ DOF_{min}[(a_1 is\ A_1)\ AND$$
$$(a_2 is\ A_2)\ AND\ (a_3 is\ A_{more\,or\,less\,intelligent})] = DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND.$$
$$(a_3 is\ A_{intelligent})] = DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND (a_3 is\ A_{very\,intelligent})]$$

On the other hand according to [2] Definition 3.1 pg.47, the *DOF* of the implication using the 'min logic operator *AND'* is equal to the *DOF* of the logic expression i.e.

$$DOF_{min}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent}\right) then\left(b\,is\,B\right)] =$$
$$DOF_{min}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent})].$$

Hence,

$$DOF_{min}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent}\right) then\left(b\,is\,B\right)]$$
$$> DOF_{min}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND(a_3 is A_{intelligent}) then\left(b\,is\,B\right)\right]$$
$$> DOF_{min}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND(a_3 is A_{very\,intelligent}) then\left(b\,is\,B\right)\right] for\ a_3 \neq 100$$

-In case of the use of the prod. Fuzzy logic operator *AND* [8],

For
$a_3 < 40\left(very\,low\,IQ\,index\right) and\,160\,a_3\left(very\,high\,IQ\,index\right)$

$$f_{prodA_{more\,or\,less\,intelligent}}\left(a_1,a_2,a_3\right) = 0\ .$$

In general

$$f_{prodA_{more\,or\,less\,intelligent}}\left(a_1,a_2,a_3\right) \geq f_{prodA_{intelligent}}\left(a_1,a_2,a_3\right) \geq f_{prodA_{very\,intelligent}}\left(a_1,a_2,a_3\right)$$
$$and\ only\ for\ a_3 = 100,\ f_{prodA\,more\,or\,less\,intelligent}\left(a_1,a_2,a_3\right) = 0.2307692308$$
$$= f_{prod\ A_{intelligent}}\left(a_1,a_2,a_3\right) = f_{prodA_{very\,intelligent}}\left(a_1,a_2,a_3\right).$$

Moreover

$$DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent})] >$$
$$DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{intelligent})] >$$
$$DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{very\,intelligent})] and\ only\ for$$
$$a_3 = 100, DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent})]$$
$$= DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND(a_3 is A_{intelligent})\ ]$$
$$= DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND(a_3 is A_{very\,intelligent})\ ].$$

On the other hand according to [2] Definition 3.1 pg.47, the *DOF* of the implication using the 'prod logic operator *AND'* is equal to the *DOF* of the logic expression i.e.

$$DOF_{prod}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent}\right)$$
$$then\left(b\,is\,B\right)] = DOF_{prod}[\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent})].$$

Hence,

$$DOF_{prod}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND\ (a_3 is A_{more\,or\,less\,intelligent}\right) then\left(b\,is\,B\right)]$$
$$> DOF_{prod}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND(a_3 is A_{intelligent}) then\left(b\,is\,B\right)\right]$$
$$> DOF_{prod}\left[If\left(a_1 is A_1\right) AND\left(a_2 is A_2\right) AND(a_3 is A_{very\,intelligent}) then\left(b\,is\,B\right)\right] for\ a_3 \neq 100$$

-Case, when the fuzzy set A_3 is equal to $A_{indeed\ intelligent}$ [8].

The membership function is

$$f_{A_{indeed\ intelligent}}(a_3) = 2 \times f^2_{A_{intelligent}}(a_3) = 2 \times f_{A_{very\ intelligent}}(a_3)$$

$$for\ f_{A_{intelligent}}(a_3) < 0.5\ i.e.\ \ a_3 \le 70\ \ or\ a_3 \ge 130,$$

$$f_{A_{indeed\ intelligent}}(a_3) = 1 - 2 \times (1 - f_{A_{intelligent}}(a_3))^2\ \ for\ 70 \le a_3 < 130,$$

and correspond to what we call indeed intelligent evaluated with IQ index see [8].

Remark that

$$f_{A_{indeed\ \ intelligent}}(a_3) = 2 \times f_{A_{very\ intelligent}}(a_3)\ \ for\ any\ a_3 \in (-\infty, 70] \cup [130, \infty)$$

$$and\ f_{A_{very\ intelligent}}(a_3) \le f_{A_{indeed\ \ intelligent}}(a_3) \le f_{A_{\ intelligent}}(a_3) \le f_{A_{\ more\ or\ less\ intelligent}}(a_3)$$ See [8] Figure 4.4.

The same Figure 4.4. Show that for $a_3 \in (70,80] \cup [120,130)$ the following inequalities hold

$$f_{A_{very\ intelligent}}(a_3) \le f_{A_{\ intelligent}} \le f_{A_{indeed\ \ intelligent}}(a_3) \le f_{A_{\ more\ or\ less\ intelligent}}(a_3)$$

And in the range $a_3 \in [80\ ,120]$ *the next inequalities hold*

$$f_{A_{very\ intelligent}}(a_3) \le f_{A_{\ intelligent}}(a_3) \le f_{A_{\ more\ or\ less\ intelligent}}(a_3) \le f_{A_{indeed\ \ intelligent}}(a_3).$$

The new fuzzy logic expression is $(a_1 is\ A_1)\ AND (a_2 is\ A_2)\ AND\ (a_3 is\ A_{indeed\ intelligent})$

And the new fuzzy logic reasoning is

$$If\ (a_1 is\ A_1)\ AND (a_2 is\ A_2)\ AND\ (a_3 is\ A_{indeed\ intelligent})\ then (b\ is\ B).$$

-In case of the use of the min. fuzzy logic operator *AND* [8], for

$$a_3 < 40 (very\ low\ IQ\ index)\ and\ 160 \le a_3 (very\ high\ IQ\ index)\ f_{\min A_{indeed\ intelligent}}(a_1, a_2, a_3) = 0.$$

In general, the following inequalities hold:

$$for\ a_3 \in (40,70]\ f_{A_{very\ intelligent}}(a_3) \le f_{A_{indeed\ intelligent}}(a_3) \le f_{A_{intelligent}}(a_3) \le f_{A_{more\ or\ less\ intelligent}}(a_3)$$

$$for\ a_3 \in (70,80]\ f_{A_{very\ intelligent}}(a_3) \le f_{A_{intelligent}} \le f_{A_{indeed\ intelligent}}(a_3) \le f_{A_{more\ or\ less\ intelligent}}(a_3)$$

$$for\ a_3 \in (80,120]\ f_{A_{very\ intelligent}}(a_3) \le f_{A_{intelligent}}(a_3) \le f_{A_{more\ or\ less\ intelligent}}(a_3) \le f_{A_{indeed\ intelligent}}(a_3)$$

$$for\ a_3 \in (130,160]\ f_{A_{very\ intelligent}}(a_3) \le f_{A_{indeed\ intelligent}}(a_3) \le f_{A_{intelligent}}(a_3) \le f_{A_{more\ or\ less\ intelligent}}(a_3)$$

Therefore, the following inequalities hold:

$$for\ a_3 \in (40,70], DOF_{min}[(a_1 is\ A_1)\ AND (a_2 is\ A_2)\ AND\ (a_3 is\ A_{more\ or\ less\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1)\ AND (a_2 is\ A_2)\ AND\ (a_3 is\ A_{intelligent})] > DOF_{min}[(a_1 is\ A_1)$$
$$AND (a_2 is\ A_2)\ AND\ (a_3 is\ A_{indeed\ intelligent})] > DOF_{min}[(a_1 is\ A_1)\ AND (a_2 is\ A_2)\ AND\ (a_3 is\ A_{very\ intelligent})]$$

$$for\ a_3 \in (70,80], DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{more\ or\ less\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{intelligent})] >$$
$$DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{very\ intelligent})]$$

$$for\ a_3 \in (80,120], DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{more\ or\ less\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{very\ intelligent})]$$

$$for\ a_3 \in (120,130]\ , DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{more\ or\ less\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{very\ intelligent})]$$

$$for\ a_3 \in (130,160]\ , DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{more\ or\ less\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent})]$$
$$> DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{very\ intelligent})]$$

On the other hand according to Definition 3.1 pg.47 [2], the *DOF* of the inference using the 'min logic operator *AND'* is equal to the *DOF* of the logic expression i.e.

$$DOF_{min}\left[\ If\ (a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent}\right)$$
$$then (b\ is\ B)] = DOF_{min}[(a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent})].$$

Hence,

For $DOF_{min}\left[\ If\ (a_1 is\ A_1) AND (a_2 is\ A_2) AND\ (a_3 is\ A_{indeed\ intelligent}\right) then (b\ is\ B)]$ the above inequalities hold.

-In case of the use of the prod. Fuzzy logic operator *AND* [8], similar results are valid.

The above results shows: in which kind the introduction of the statement

$(a_3 is\ A_{intelligent})$,$(a_3 is\ A_{very\ intelligent})$, $(a_3 is\ A_{more\ or\ less\ intelligent})$, $(a_3 is\ A_{indeed\ intelligent})$ lied to the changes of the *DOF - s* of the logical expressions and that of the reasoning's in sense of Definition [2] 3.1. pg.47.

8. The effect of introduction in premises of the statement *($a3$ is $A_{intelgent}$)* *concerning* the changes of the values of conditional possibility distribution

The case of minimum logic, and product logic operator *AND*, using minimum and product t norm respectively is discussed; see section 2 of the present paper. The example is the same; 'If it is cold and I have a long way to walk, then I usually take my coat'.

$If\ the\ min\ logic\ operator\ AND\ with\ the\ minimum\ t\ norm\ are\ selected\ then\ for\ u=(5\ ,500,50)\ and\ v=0.25$

$we\ have: DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)].=\min[0.2307692308,$

$f_{A_3}(a_3)]=\min[0.2307692308, 0.1666666667]=0.1666666667$ and $f_B(v)=0.5$.

Therefore

$DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]$

$\le f_B(v)\ and\ the\ conditional\ possibility\ distribution\ \Pi^{min}_{B\nmid A}\ is\ equal\ to\ 1, \Pi^{min}_{B\nmid A}(u,v)=1$

$If\ the\ min\ logic\ operator\ AND\ with\ the\ minimum\ t\ norm\ are\ selected\ then\ for\ u=(5\ ,500,50)\ and\ v=0.99$

$we\ have: DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)].=\min[0.2307692308,$

$f_{A_3}(a_3)]=\min[0.2307692308,0.1666666667]=0.1666666667$ and $f_B(v)=0.02.$

Therefore

$DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]> f_B(v)\ and\ the\ conditional$

$possibility\ distribution\ \Pi^{min}_{B\nmid A}\ is\ equal\ to\ f_B(v)=0.02, \Pi^{min}_{B\nmid A}(u,v)=0.02$

$If\ the\ min\ logic\ operator\ AND\ with\ the\ the\ product\ t\ norm\ are\ selected\ then\ for\ u=(5\ ,500,50)\ and\ v=0.25$

$we\ have: DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)].=\min[0.2307692308,$

$f_{A_3}(a_3)]=\min[0.2307692308, 0.1666666667]=0.1666666667$ and $f_B(v)=0.5$.

Therefore

$DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]$

$< f_B(v)\ and\ the\ conditional\ possibility\ distribution\ \Pi^{min}_{B\nmid A}$

$=\min\left\{1,\frac{0.5}{0.1666666667}\right\}=\min\{1,2.999999999\}=1, is\ equal\ to\ \Pi^{min}_{B\nmid A}=1.$

$If\ the\ min\ logic\ operator\ AND, with\ the\ the\ product\ t\ norm\ are\ selected\ then\ for\ u=(5\ ,500,50)\ and\ v=0.99$

$we\ have: DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)].=\min[0.2307692308,$

$f_{A_3}(a_3)]=\min[0.2307692308,0.1666666667]=0.1666666667$ and $f_B(v)=0.02.$

Therefore

$DOF_{min}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]> f_B(v)\ and\ the\ conditional\ possibility\ distribution$

$\Pi^{min}_{B\nmid A}=\min\left\{1,\frac{0.02}{0.1666666667}\right\}=\min\{1,0.1200000000\}=0.1200000000\ is\ equal\ to\ \Pi^{min}_{B\nmid A}=0.1200000000$.

$If\ the\ prod\ logic\ operator\ AND, with\ the\ the\ minimum\ t\ norm\ are\ selected\ then\ for$

$u=(5\ ,500,50)\ and\ v=0.25\ we\ have: DOF_{prod}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND$

$(a_3 is\ A_3)].=0.2307692308\times f_{A_3}(a_3)]=0.2307692308\times 0.1666666667]=0.03846153847$ and $f_B(v)=0.5.$

$Therefore\ DOF_{prod}[(a_1 is\ A_1)\ AND(a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]< f_B(v)$

$and\ the\ conditional\ possibility\ distribution\ \Pi^{min}_{B\nmid A}(u,v)\ is\ equal\ to\ 1, \Pi^{min}_{B\nmid A}(u,v)=1$.

If the prod logic operator AND, with the the minimum t norm are selected then for

$u = (5\ ,500,50)$ *and* $v = 0.99$ *we have* : $DOF_{prod}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_3)].$

$= 0.2307692308 \times f_{A_3}(a_3)] = 0.2307692308 \times 0.1666666667] = 0.03846153847$ and $f_B(v) = 0.02.$

Therefore $DOF_{prod}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]$

$> f_B(v)$ *and the conditional possibility distribution*

$\Pi^{min}_{B \nmid A}(u,v)$ *is equal to* $f_B(v), \Pi^{min}_{B \nmid A}(u,v) = 0.02$.

If the prod logic operator AND with the the product t norm are selected then for $u = (5\ ,500,50)$ *and* $v = 0.25$

we have : $DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_3)]. = 0.2307692308 \times$

$f_{A_3}(a_3)] = 0.2307692308 \times 0.1666666667 = 0.03846153847$ and $f_B(v) = 0.5$.

Therefore $DOF_{prod}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_3)] < f_B(v)$ *and the conditional*

possibility distribution $\Pi^{min}_{B \nmid A} = \min\left\{1, \dfrac{0.5}{0.03846153847}\right\} = \min\{1,13\} = 1,$ *is equal to* $\Pi^{min}_{B \nmid A} = 1.$

If the prod logic operator AND with the the product t norm are selected then for

$u = (5\ ,500,50)$ *and* $v = 0.99$ *we have* : $DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_3)].$

$= 0.2307692308 \times f_{A_3}(a_3)] = 0.2307692308 \times 0.1666666667 = 0.03846153847$ and $f_B(v) = 0.02$.

Therefore $DOF_{min}[(a_1 is\ A_1)\ AND\ (a_2 is\ A_2)\ AND\ (a_3 is\ A_3)] < f_B(v)$ *and the conditional possibility distribution*

$\Pi^{min}_{B \nmid A} = \min\left\{1, \dfrac{0.02}{0.03846153847}\right\} = \min\{1, 0.5199999999\} = 0.5199999999,$ *is equal to* $\Pi^{min}_{B \nmid A} = 0.5199999999.$

These results show that the introduction of the statement (a_3 is $A_{intelligenr}$) could have a significant effect on the conditional possibility distribution value i.e. the *DOF – s of fuzzy logic reasoning*.

9. Results

Computation developed in the framework of 'human intelligence' linguistic variable reveal: the meanings of the fuzzy logic operator *'IF...THEN'*; the dependence of the 'possible true value' of the implication 'IF......THEN' on the considered meanings; the fuzzy logic reasoning structure; the concepts of minimum maximum and additive combinations; the meaning of deffuzzyfication; the effect of the incorporation of different fuzzy concepts of the 'human intelligence' linguistic variable into the premises of fuzzy reasoning.

10. Comments and conclusion

In medical sciences experts rely on empirical knowledge and experience in diagnosis and treatment of diseases. There are no generally accepted strict laws expressed in precise mathematical form as in 'hard disciplines 'such as physics. Such 'soft' disciplines are ideal for applying fuzzy methods. The fuzzy-rule based approach is applied to arterial hypertension and addresses questions such as disease severity, indication of etiological check-up, hospitalization, coronary risk, and indication of antihypertensive treatment. These are questions that a physician may have to answer regarding any hypertensive patient. In the experiment described in detail in [10] five medical experts in the hypertension field have provided their answers to five questions for one hundred patient files, case by case. The stated aim of reproducing the (average) opinion of five experts was achieved, yielding highly encouraging results. Fuzzy reasoning in the framework of the 'human intelligence' fuzzy linguistic variable, developed in this paper reveal a large scale of understanding the true value of fuzzy reasoning and make it possible that within this framework "severe" and "moderate" pathology may be both be "true "for a given patient.

Authors' contribution

Both authors contributed equally to the development of this work. All authors have read and agreed to the published version of the manuscript.

Data availability statement

The original contributions presented in the study are included in the article; further information can be requested from the corresponding author.

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