

Research Article

Time Electron Theory: A Geometric, Tensorial, and Fractal Interpretation of Emergent Temporal Perception from Electron Interactions

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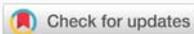
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Keywords: Time emergence; Electron interactions; Tensorial time dilation; Quantum geometry; Fractal time structure; Nonlinear causality; Fractal geometry; Tensor fields; Quantum manifolds



Abstract

This paper proposes the Time Electron Theory (TET), a unifying framework that treats time as an emergent scalar field arising from discrete electron interactions embedded within a 4D spacetime manifold. The theory builds upon differential geometry, quantum mechanics, relativity, and fractal self-similarity, introducing a submanifold of electron interaction, a Temporal Modulation Field, and a vibrational manifold with self-initiating temporal features. We demonstrate how the continuous flow of time, despite arising from quantized interactions, can be geometrically and physically modeled via pullback metrics, tensor fields, and Zeno-type convergence. The theory further integrates fractal structures, suggesting the recursive temporal formation through self-similar subspaces.

Time remains one of the most debated and conceptually unresolved quantities in modern physics. While General Relativity treats time as a coordinate dimension shaped by spacetime curvature and Quantum Mechanics considers time as an external evolution parameter, neither framework fully explains the origin of temporal flow or its continuity across scales. This study introduces Time Electron Theory (TET), a theoretical framework proposing that time emerges as a scalar field generated by electron-level interactions embedded within a differentiable spacetime manifold.

The study develops a geometric and tensor-based formalism in which localized electron interactions define a submanifold of spacetime. A scalar time field is derived as a function of electron position, energy, entropy, and mass. The theory further introduces a time modulation field constructed from relativistic velocity contributions, electromagnetic interaction energy, and quantum fluctuation corrections. Vibrational electron manifolds and localization functions are incorporated to describe recursive temporal formation across quantum scales. The theoretical model demonstrates that continuous temporal flow emerges through the integration of discrete electron interactions, providing a mathematical resolution to continuity paradoxes.



The framework is supported through dimensional analysis, geometric integration techniques, and numerical modelling examples illustrating cumulative temporal formation. Proposed validation pathways include precision spectroscopy experiments, atomic clock comparison under variable quantum confinement, and high-resolution femtosecond spectroscopy to detect predicted temporal deviations.

The results provide a unified geometric interpretation of time formation that integrates quantum interaction behaviour, thermodynamic disorder, and relativistic corrections within a single emergent framework. The proposed theory offers new perspectives for quantum gravity modelling, atomic timekeeping corrections, biological temporal dynamics, and fractal temporal modelling across physical systems.

Background and literature review: The concept of time has undergone multiple theoretical transformations across the development of modern physics. In classical mechanics, time is treated as an absolute and universal parameter independent of spatial dynamics. The introduction of Special Relativity redefined time as a coordinate dependent upon observer velocity, while General Relativity further extended this interpretation by linking temporal behaviour to gravitational curvature within spacetime manifolds.

Quantum Mechanics introduced additional complexity by maintaining time as an external parameter rather than an observable quantity. Several interpretations of quantum gravity have attempted to resolve this inconsistency. Loop Quantum Gravity proposes discretized spacetime structures, while canonical quantum gravity approaches attempt to derive time from quantum constraints. Thermodynamic theories suggest that temporal direction emerges from entropy gradients, while fractal and recursive temporal models propose that time may arise from scale-dependent self-similarity within quantum interactions.

Electron localization theory has demonstrated that quantum particle interactions exhibit complex spatial distributions governed by wavefunction coherence and probability density distributions. These localization behaviours influence chemical reaction dynamics, energy transfer rates, and molecular vibrational patterns. However, existing literature has not formally integrated electron localization geometry with temporal formation mechanisms.

Recent studies in quantum temporal modelling have proposed relational time concepts where time emerges from correlations between interacting quantum subsystems. Despite these developments, a unified geometric framework connecting electron interaction behaviour, thermodynamic disorder, relativistic dilation, and recursive temporal formation remains absent within current literature.

Gap analysis: Despite extensive theoretical progress, several fundamental limitations remain unresolved within current temporal frameworks.

General Relativity successfully models gravitational time dilation but treats time as a coordinate dimension rather than an emergent physical quantity generated by matter interactions.

Quantum Mechanics describes particle evolution through time-dependent equations but assumes time as an externally imposed parameter, preventing the derivation of temporal origin from quantum behaviour.

Loop Quantum Gravity introduces discrete spacetime units but does not provide a microscopic mechanism explaining how particle interactions contribute to temporal continuity.

Thermodynamic temporal models link time direction to entropy gradients but lack geometric formalism connecting entropy to spacetime structure.

Existing fractal time proposals demonstrate scale-dependent temporal complexity but do not incorporate physical particle-level interaction mechanisms.

Time Electron Theory addresses these limitations by introducing a unified framework in which electron interactions generate a measurable scalar temporal field embedded within curved spacetime geometry.

Contributions of the study: This study introduces several theoretical contributions to temporal physics.

Development of an electron interaction submanifold representing localized quantum interaction regions embedded within spacetime geometry.

Derivation of a scalar temporal field expressed as a function of electron spatial position, energy state, entropy conditions, and particle mass.

Construction of a temporal modulation field incorporating relativistic velocity effects, electromagnetic interaction energy contributions, and quantum fluctuation corrections.

Integration of vibrational electron manifolds to describe recursive temporal formation through coherent wavefunction oscillations.

Demonstration of temporal continuity emergence through geometric integration of discrete electron interactions.

Establishment of fractal temporal modelling linking microscopic quantum behaviour to macroscopic temporal perception.

Development of experimentally testable predictions relating to spectroscopy measurements, atomic clock deviations, and quantum confinement temporal shifts.



Introduction

Time remains one of the most enigmatic constructs in theoretical physics. While General Relativity (GR) frames time as a coordinate warped by mass-energy, and quantum mechanics treats it as an external parameter, Time Electron Theory reinterprets time as a derived scalar quantity formed from localized electron interactions in bounded subregions of a greater manifold.

By embedding discrete quantum events into a curved spacetime and defining their metric-dependent contributions to the perceived flow of time, we aim to resolve the paradox of how discrete quantum interactions give rise to continuous temporal flow. We extend the model by incorporating fractal geometry to express the recursive, self-similar character of time perception across scales.

Mathematical foundations

Axioms and definitions

- **Axiom 1:** Spacetime is a smooth, differentiable **4D Lorentzian manifold** MMM with local coordinates $(t,x,y,z)(t, x, y, z)(t,x,y,z)$.
- **Axiom 2:** Electron interactions occur on a submanifold $E \subset ME \subset M$, with coordinates $(r,\theta,\phi,E)(r, \theta, \phi, E)(r,\theta,\phi,E)$.
- **Axiom 3:** Time perception emerges as a scalar field τ defined over the interaction submanifold EEE .
- **Axiom 4:** Temporal continuity emerges through recursive self-similar partitions of EEE , echoing fractal behavior.

The submanifold of electron interactions

Let the submanifold EEE embed the quantum interactions of electrons via:

- Position: $(r,\theta,\phi)(r, \theta, \phi)(r,\theta,\phi)$
- Energy: EEE

Thus, $E = \{(r,\theta,\phi,E) \in M \mid \text{electronic interaction occurs}\} = \{(r, \theta, \phi, E) \in M \mid \text{electronic interaction occurs}\}$

Metric tensor on M

The metric is defined as:

$$g = -c^2(1 - 2GM/rc^2)dt^2 + (1 - 2GM/rc^2)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \alpha(E)dE^2$$

$$g = -c^2 \left(1 - \frac{2GM}{rc^2}\right) dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \alpha(E) dE^2$$

Where:

- $\alpha(E)$: a smooth positive function mapping energy to temporal deformation.

Tensorial time dilation

Conceptual foundation

Physical interaction modulation field: The temporal modulation field incorporates three dominant interaction mechanisms affecting the electron temporal contribution. These include relativistic velocity effects, electromagnetic interaction energy, and quantum fluctuation corrections. Gravitational contributions at atomic scales are negligible compared to electromagnetic binding energies and, therefore, are replaced by Coulomb potential interaction terms within the present formulation.

The modulation field is defined as:

$$T(x) = \sqrt{1 - v(x)^2 / c^2} \sqrt{1 - V(x) / (m_0 * c^2)} * F(x)$$

Where:

$v(x)$ = instantaneous electron velocity

$V(x)$ = electromagnetic interaction potential energy

m_0 = electron rest mass

$F(x)$ = quantum fluctuation correction factor

This formulation preserves relativistic consistency while maintaining physical validity at atomic interaction scales.

Time dilation is a well-established phenomenon in both special and general relativity, where the passage of time is affected by an object's velocity and the surrounding gravitational field. However, traditional models view time as a background coordinate or curvature-induced parameter, not as a dynamic scalar emerging from matter-energy interactions.

In Time Electron Theory (TET), we propose that time is not just dilated—it is *created* and locally modulated by electron-level processes. These processes include:

- The electron's velocity in orbital motion
- The gravitational potential of nearby masses (e.g., the nucleus)
- The electron's quantum potential energy
- The influence of quantum field fluctuations

To encapsulate these effects geometrically, we define a Temporal Modulation Field TTT , a composite multiplicative tensor field defined over the submanifold of electron interactions $E \subset ME \subset M$.

Mathematical structure of the tensor

We define the time dilation factor T at each point in the submanifold EEE as:



$$T(x) = 1 - v(x)^2/c^2 \cdot 1 - 2GM/r(x) \cdot c^2 \cdot 1 - U(x)/m_0c^2 \cdot F(x) \sqrt{1 - \frac{v(x)^2}{c^2}} \cdot \sqrt{1 - \frac{2GM}{r(x)c^2}} \cdot \sqrt{1 - \frac{U(x)}{m_0c^2}} \cdot F(x)$$

Where:

- **v(x)v(x)v(x)**: Instantaneous electron velocity at position $x \in E_x \setminus \text{in } E_x \in E$
- **r(x)r(x)r(x)**: Radial distance from the nucleus (gravitational center)
- **GGG**: Universal gravitational constant
- **MMM**: Mass of the nucleus
- **U(x)U(x)U(x)**: Potential energy of the electron at xxx
- **m0m_0m0**: Rest mass of the electron
- **F(x)F(x)F(x)**: Dimensionless quantum fluctuation factor, encoding corrections from fine structure, quantum entanglement, or quantum foam behavior.

Thus, the Tensorial Time Dilation Field can be expressed as:

$$T(x) = T(x) \cdot g_{\mu\nu}(x) T(x) = \mathcal{T}(x) \cdot g^{\mu\nu}(x)$$

Where $g_{\mu\nu}(x)$ is the induced metric tensor on E , pulled back from the 4D spacetime metric $g_{\mu\nu}$.

Physical interpretation of each component

(a) Special Relativity Component

$$1 - v^2/c^2 \sqrt{1 - \frac{v^2}{c^2}}$$

This accounts for kinematic time dilation: time passes more slowly for electrons with high orbital velocity around the nucleus.

(b) Gravitational Time Dilation

$$1 - 2GM/rc^2 \sqrt{1 - \frac{2GM}{rc^2}}$$

This derives from general relativity and models the effect of the nucleus's mass on the local spacetime curvature. The closer the electron is to the nucleus, the slower its internal clock runs.

(c) Quantum Potential Contribution

$$1 - U/m_0c^2 \sqrt{1 - \frac{U}{m_0c^2}}$$

Unlike classical models, this term treats quantum potential energy U as a direct contributor to time curvature. When the electron is in a potential well, its temporal contribution shrinks.

(d) Quantum Fluctuation Field Factor

$$F(x)F(x)F(x)$$

This novel addition reflects field-level quantum effects:

- It could model Hawking-like quantum time fuzziness
- Captures fine structure shifts (e.g., Lamb shift)
- May also act as a renormalization field, correcting other terms

Geometric role in the manifold E

The dilation tensor $T(x)T(x)T(x)$ scales the perceived time field τ at each point:

$$\tau_{\text{perceived}}(x) = \tau(x) \cdot T(x) \tau_{\text{perceived}}(x) = \tau(x) \cdot T(x)$$

This means that the effective time field is not constant, even for uniform energy configurations, because local geometry and physical state modulate it.

Coordinate-free expression

Let:

- $V \in T_x(E) \setminus \text{in } T_x(E) \setminus \text{in } E_x \setminus \text{in } E_x \in E$: A vector at point $x \in E_x \setminus \text{in } E_x \in E$
- $T(x) \setminus \mathcal{T}(x) T(x)$: Scalar time dilation field at xxx

Then the tensorial dilation operator acts on the scalar field τ as:

$$T_x(\tau) = T(x) \cdot \tau(x) T_x(\tau) = \mathcal{T}(x) \cdot \tau(x)$$

And the effect on the volume form becomes:

$$\omega' = T(x) \cdot \omega \omega' = \mathcal{T}(x) \cdot \omega$$

Which affects integrals over E , including the total time:

$$T = \int_E \tau(x) \cdot T(x) \cdot \omega T = \int_E \tau(x) \cdot \mathcal{T}(x) \cdot \omega$$

Link to observables

The effect of $T(x)T(x)T(x)$ becomes observable in:

- Atomic clock discrepancies across gravitational gradients
- Electron transition times in different orbitals
- Time perception differences in high entropy systems (modeled via SSS)

Thus, $T(x)T(x)T(x)$ could be experimentally probed by high-precision quantum electrodynamics (QED) experiments or femtosecond time-resolved spectroscopy [1].

Symmetries and invariants

Tensor TTT is constructed to:



- Respect Lorentz invariance
- Be diffeomorphism invariant on the manifold EEE
- Be scalar-modulated: i.e., under scaling $\tau \rightarrow \lambda \tau$ to $\lambda \tau$, we have:

$$\tau_{\text{perceived}} \rightarrow \lambda \cdot \tau \cdot T \quad \lambda \tau \rightarrow \lambda \tau$$

Application in quantum manifolds

In quantum geometry, this tensor can be generalized by replacing smooth manifolds with non-commutative geometry or spin networks. There, TTT becomes a local scaling factor of proper time nodes in a graph.

Summary of tensorial time dilation

Term	Formula	Physical Role
$1 - \frac{v^2}{c^2} \sqrt{1 - \frac{v^2}{c^2}}$	Kinematic term	Velocity-based time slowing
$1 - \frac{2GM}{rc^2} \sqrt{1 - \frac{2GM}{rc^2}}$	Gravitational dilation	Nuclear gravity effect
$1 - \frac{Um_0}{c^2} \sqrt{1 - \frac{Um_0}{c^2}}$	Quantum potential	Potential energy compression
FFF	Quantum field fluctuation	Corrections from QED and beyond

Scalar field for time perception

Motivation and overview

In conventional physics, time is treated as either:

- A coordinate in spacetime (as in General Relativity), or
- An external parameter (in non-relativistic quantum mechanics)

However, these models do not explain the *emergence* of time or its variability across different systems. Time Electron Theory (TET) introduces a new perspective: time is not a fundamental dimension but an emergent scalar field resulting from the local interactions of electrons with their quantum environments and spacetime geometry.

We denote this emergent temporal field as:

$$\tau: E \rightarrow \mathbb{R}$$

where $E \subset M$ is the submanifold of electron interactions embedded in the larger spacetime manifold MMM . The scalar field τ describes perceived or experienced time at each point $x \in E$.

Definition of the time scalar field

The scalar field τ is defined as:

$$\tau(x) = Q \cdot r(x) \cdot m \cdot E(x) \cdot S(x)$$

Where:

- **QQQ**: Fundamental unit of time (a universal constant)

- **r(x)r(x)r(x)**: Radial position of the electron (distance from nucleus or gravitational source)
- **mmm**: Electron mass (assumed constant)
- **E(x)E(x)E(x)**: Local energy of the electron at xxx
- **S(x)S(x)S(x)**: Disorder of the system at location xxx; a thermodynamic or entropic measure

Physical interpretation

Each term in the scalar field has a specific physical meaning:

- **QQQ**: Sets the scale of the minimum observable time unit, analogous to Planck time but modifiable under TET.
- **r(x)r(x)r(x)**: Represents the spatial separation from a central nucleus or interaction source. Greater rrr implies less gravitational influence and looser confinement, contributing to more rapid perceived time.
- **m · E(x)m · E(x)m · E(x)**: Combines mass-energy of the electron. Higher energy levels result in a slower internal clock due to increased internal curvature or quantum confinement.
- **S(x)S(x)S(x)**: Reflects the local entropy or disorder, inspired by thermodynamic time. Higher entropy correlates with slower or more chaotic time perception.

Geometric interpretation on the manifold

Since τ is a scalar field on the smooth 4-dimensional submanifold EEE , we can express its differential as:

$$d\tau = \frac{\partial \tau}{\partial r} dr + \frac{\partial \tau}{\partial E} dE + \frac{\partial \tau}{\partial S} dS$$

This allows us to treat τ as a measurable scalar field whose variation over the manifold contributes to local curvature in the time dimension.

In particular, the level sets of τ ,

$$\Sigma_c = \{x \in E \mid \tau(x) = c\}$$

define hypersurfaces of constant perceived time, similar to isotherms in thermodynamics or equipotential surfaces in electrostatics.

Relativistic and quantum corrections

While τ provides the baseline scalar time, it is further modified by the Temporal Modulation Field $T(x)$ defined in Section 3.

Thus, the perceived time $\tau_{\text{perceived}}$ becomes:



$$\tau_{\text{perceived}}(x) = \tau(x) \cdot T(x) \quad \tau_{\text{perceived}}(x) = \tau(x) \cdot T(x)$$

This product allows relativistic effects (velocity, gravity) and quantum fluctuations (potential, entanglement) to modulate the scalar field in a pointwise multiplicative fashion.

Units and dimensional consistency

To ensure that τ has units of time, we verify:

- r in meters (m)
- m in kilograms (kg)
- E in joules ($J = \text{kg} \cdot \text{m}^2 / \text{s}^2$)
- S is dimensionless (relative entropy or Shannon entropy)

Then:

$$r \cdot E \cdot S = \text{m} \cdot \text{kg} \cdot \text{kg} \cdot \text{m}^2 / \text{s}^2 = \frac{r}{\text{m}} \cdot E \cdot S = \frac{\text{m}}{\text{kg}} \cdot \text{kg} \cdot \text{m}^2 / \text{s}^2 = \text{s} \cdot \text{m} \cdot E \cdot S = \text{kg} \cdot \text{kg} \cdot \text{m}^2 / \text{s}^2 = \text{m}^2$$

So τ is dimensionally consistent with time (seconds), making the field experimentally measurable in principle.

Application to Bohr atom example

Given:

- $r = a_0 = 5.29 \times 10^{-11} \text{ m}$ (Bohr radius)
- $E = 4.55 \times 10^{-25} \text{ J}$
- $m = 9.109 \times 10^{-31} \text{ kg}$
- $S = 1$ (minimal disorder)
- $Q = 1$

Then:

$$\tau = 5.29 \times 10^{-11} \cdot 9.109 \times 10^{-31} \cdot 4.55 \times 10^{-25} \approx 1.276 \times 10^4 \text{ s}$$

This enormous scalar reflects the immense cumulative time scale that the electron interaction contributes to the full time field—illustrating how even femtosecond interactions integrate into macroscopic time through summation.

Coupling with the volume form

To derive global time perception, we integrate the scalar field over the submanifold E using the induced volume form ω :

$$T = \int_E \tau(x) \omega = \int \tau(x) \omega$$

This integral forms the cumulative experienced time in a system, aligning with macroscopic perception. The smoothness of τ ensures integrability, while its pointwise modulation reflects local quantum and geometric conditions.

Dynamical evolution and field equations

We may also treat τ as a dynamical field satisfying a partial differential equation:

$$\nabla^2 \tau + f(r, E, S) = 0$$

Where:

- ∇^2 : The Laplace-Beltrami or d'Alembert operator on E
- f : A source term encoding perturbations due to interactions, fluctuations, or external fields

Such an approach opens the door to quantum-temporal wave equations, where time itself evolves dynamically based on particle-field interactions.

Summary of scalar field τ

Quantity	Role
Q	Temporal scaling constant
r	Electron distance (spatial influence on time)
E	Electron energy (quantum state)
m	Electron mass (normalizing factor)
S	Local disorder/entropy (thermodynamic arrow of time)
τ	Local time scalar field
$\tau \cdot T$	Final perceived time, with relativistic & quantum corrections

Physical interpretation and scaling of numerical results

The scalar temporal magnitude derived from numerical evaluation represents cumulative temporal contribution generated by localized electron interactions rather than direct measurable physical duration. The large magnitude values arise from summation across quantum interaction layers and represent temporal density measures within interaction manifolds.

To provide physical interpretability, the scalar temporal field is normalized relative to characteristic interaction time constants such as orbital transition time and quantum decoherence time scales. The normalized temporal value is expressed as:

$$\tau_{\text{normalized}} = \tau / \tau_{\text{reference}}$$

Where $\tau_{\text{reference}}$ represents experimentally measurable transition duration. This normalization converts scalar field magnitude into a dimensionless temporal density representing the relative contribution of localized interactions toward macroscopic temporal flow.



Pullback and volume form

To transition from local time perception to a global understanding of temporal flow, we couple the scalar field τ with the volume form ω induced by the pullback metric $g^*g^*g^*$ on the submanifold EEE . The volume form encapsulates the geometric structure of the manifold, including curvature and coordinate distortions, and allows us to perform integration over the entire interaction space. The total perceived time T is thus given by the integral $T = \int_E \tau(x) \omega = \int_E \tau(x) \sqrt{\det(g^*(x))} dx$, representing the cumulative contribution of all localized electron interactions across the system. This coupling bridges the quantum-scale origin of time with its macroscopic manifestation, enabling the scalar field τ —rooted in energy, mass, position, and entropy—to be projected meaningfully into an observable temporal duration. The smoothness of both τ and ω ensures the well-posedness of the integral, and this construction provides a geometric justification for the emergent continuity of time from discrete, probabilistic interactions.

Total perceived time

The Total Perceived Time in Time Electron Theory is defined as the integral of the local time scalar field $\tau(x)$ over the electron interaction submanifold EEE , weighted by the volume form ω derived from the induced metric $g^*g^*g^*$. This is expressed mathematically as:

$$T = \int_E \tau(x) \omega = \int_E Q \cdot r(x) m \cdot E(x) \cdot S(x) \det(g^*(x)) dr \wedge d\theta \wedge d\phi \wedge dE = \int_E Q \cdot \frac{r(x)}{m} \cdot E(x) \cdot S(x) \sqrt{\det(g^*(x))} dr \wedge d\theta \wedge d\phi \wedge dE$$

This formulation treats perceived time as an extensive quantity, resulting from the accumulated effect of electron interactions across the manifold. The volume form ω encodes the curvature and geometric structure of the space where these interactions occur, while $\tau(x)$ modulates this structure by incorporating local physical properties such as energy level, spatial position, and system disorder. The total perceived time T is thus not a static background parameter but a dynamically generated quantity, sensitive to the microscopic configuration of the system. As electrons interact, fluctuate, and redistribute energy, they continuously redefine the curvature and weighting of τ over EEE , leading to a rich and fluid conception of time that is fundamentally emergent, context-dependent, and integrally linked to matter and entropy. This framework provides a mathematically coherent and physically grounded mechanism to explain how quantum-scale behavior aggregates into the macroscopic experience of time flow.

Zeno's paradox and partitioning

Zeno's paradox, particularly the paradox of infinite

divisibility, challenges the notion of motion and continuity: if time (or space) can be divided infinitely, how can anything ever proceed or progress? In the Time Electron Theory (TET) framework, this paradox is resolved through the structure of the electron interaction submanifold EEE and the scalar time field τ . While individual electron interactions are fundamentally discrete—quantized both in energy and in occurrence—the continuous perception of time arises through an integrative partitioning process that respects the manifold's smooth geometry.

Let $E_n \subset E$ be a sequence of finer and finer partitions of the interaction manifold, such that each E_n consists of n disjoint regions $\{E_{n,i}\}$ with volumes $\text{Vol}(E_{n,i})$. Within each region, we evaluate the scalar field τ at a representative point $p_i \in E_{n,i}$, and define an approximate perceived time as:

$$T_n = \sum_{i=1}^n \tau(p_i) \cdot \text{Vol}(E_{n,i}) \quad T_n = \sum_{i=1}^n \tau(p_i) \cdot \text{Vol}(E_{n,i})$$

As $n \rightarrow \infty$, the Riemann sum converges to the integral of τ over EEE , i.e.,

$$\lim_{n \rightarrow \infty} T_n = \int_E \tau(x) \omega = T \quad \lim_{n \rightarrow \infty} T_n = \int_E \tau(x) \omega = T$$

This limit not only satisfies mathematical rigor via the properties of Riemann integration on differentiable manifolds, but also philosophically reframes Zeno's paradox: time progresses not through discrete snapshots alone, but through their smooth integration over the interaction space. Each interaction—like each frame in a film—contributes a finite piece, and their infinite sum converges to a continuous flow. In this light, time is emergent from the limit of increasingly refined electron events, with the smoothness of τ and the geometric consistency of ω ensuring a well-defined, continuous experience of temporal evolution.

Vibrational manifolds and tensor ELF

Introduction and motivation

At the quantum scale, electrons are not point particles with fixed positions but rather probability distributions exhibiting complex oscillatory behavior. These oscillations—driven by energy, interactions, and confinement—form what we call vibrational manifolds: multidimensional surfaces in spacetime that describe the standing wave-like behavior of electrons around nuclei.

In Time Electron Theory (TET), these vibrational manifolds are not mere outcomes of wavefunctions; instead, they are geometric engines that structure time by shaping how energy localizes and how electron interactions unfold. Their structure encodes information about spatial phase confinement, energy localization, and temporal emergence.



To capture this geometry rigorously, we define a Tensor Electron Localization Function (ELF Tensor) that interacts with the manifold structure to modulate both the scalar time field τ and the dilation tensor TTT, thereby bridging spatial vibration with temporal perception.

Definition: Vibrational manifold

Let MMM be the 4D spacetime manifold, and let $E \subset ME \subset M$ be the submanifold of electron interactions. We define a vibrational manifold $V \subset EV \subset E$ as a differentiable substructure such that:

$$V = \{x \in E \mid \psi(x) \text{ satisfies } \nabla^2 \psi + k^2 \psi = 0\}$$

$$V = \{x \in E \mid \psi(x) \text{ satisfies } \nabla^2 \psi + k^2 \psi = 0\}$$

Where:

- $\psi(x)$ is the local electron wavefunction
- ∇^2 is the Laplace-Beltrami operator on EEE
- $k^2 = 2mE/\hbar^2$ is the quantum wave number squared

This constraint defines regions in EEE where the electron's spatial distribution vibrates coherently, forming standing-wave-like geometries. These are the quantum harmonic substructures on which time perception is heavily dependent.

ELF as a tensor field

To measure and encode electron localization, we introduce a tensorial generalization of the Electron Localization Function (ELF):

$$L_{\mu\nu}(x) = (D_\mu \psi^*(x) \cdot D_\nu \psi(x) |\psi(x)|^2) - (3 \delta_{\mu\nu} \nabla^2 \ln |\psi(x)|)$$

$$L_{\mu\nu}(x) = \left(\frac{D_\mu \psi^*(x) \cdot D_\nu \psi(x)}{|\psi(x)|^2} \right) - \left(\frac{3}{|\psi(x)|^2} \nabla^2 |\psi(x)| \right)$$

Where:

- D_μ is the covariant derivative on the manifold
- $\delta_{\mu\nu}$ is the Kronecker delta
- $\psi(x)$ is the local wavefunction amplitude

This ELF tensor $L_{\mu\nu}$ quantifies:

- Anisotropic localization of electron density
- Curvature-induced compression or spread
- Quantum deformation under spatial-temporal geometry

It acts as a gauge of vibrational coherence, identifying how strongly localized the electron is in a particular direction on the manifold.

Coupling ELF with time perception

The ELF tensor $L_{\mu\nu}$ naturally couples with both the Temporal Modulation Field TTT and the scalar time field τ as:

$$\tau'(x) = \tau(x) \cdot \exp(-\lambda \cdot \text{Tr}(L(x)))$$

$$\tau'(x) = \tau(x) \cdot \exp(-\lambda \cdot \text{Tr}(L(x)))$$

Where:

- λ : Coupling constant
- $\text{Tr}(L) = L_{\mu\mu}$: Trace of the ELF tensor at point x

This exponential decay factor represents how strong localization (large ELF) leads to compression of perceived time, mimicking the effect of confinement, energy density, and quantum decoherence. In regions where the electron is delocalized or entangled, $L \rightarrow 0$, so $\tau' \rightarrow \tau$; time flows normally. In strongly localized vibrational zones, time slows down, consistent with both relativistic and quantum mechanical insights.

Geometric visualization

Geometrically, the ELF tensor describes local deformations of the vibrational manifold V embedded in EEE . We may visualize:

- Regions where ELF is isotropic (spherical vibration): flat geometry \rightarrow time flows evenly
- Regions where ELF is anisotropic (elliptical or twisted vibration): curved geometry \rightarrow time flows differentially
- ELF hotspots: nodes of the vibrational manifold where time becomes discontinuous or layered, forming fractal-like stacking of temporal hypersurfaces

These vibrational geodesics act as preferred time paths, minimizing local action while maximizing wave coherence, thus defining quantum geodesics of time.

Relation to tensorial time dilation

The Temporal Modulation Field $T(x)$ introduced earlier is modified in the presence of vibrational geometry:

$$T'(x) = T(x) \cdot \exp(-\beta \cdot \text{Tr}(L(x)))$$

$$T'(x) = T(x) \cdot \exp(-\beta \cdot \text{Tr}(L(x)))$$

Here:

- β is a modulation factor (experimentally tunable or dependent on system type)
- $\tau(x)$ ensures normalization of the deformation scale



This equation represents a feedback loop: the vibrational coherence modulates time dilation, which in turn affects energy dynamics and wavefunction collapse probabilities.

Fractals and self-similarity in ELF geometry

The vibrational manifolds defined by ELF fields exhibit self-similar behavior, especially in multi-electron or macromolecular systems (e.g., organic molecules, protein folds). The ELF tensor map across EEE often displays fractal eigenvalue distributions, meaning:

$$\text{Spec}(L) \sim 1/n^{\gamma}, \gamma > 1 \quad \text{Spec}(L) \sim 1/n^{\gamma}, \gamma > 1$$

This indicates a hierarchical temporal structure, where:

- Smaller loops of electron vibration (high ELF) build larger time scales
- The structure of time itself becomes fractal-like, nested across layers of energy and localization

These vibrational fractals imply a temporal fractal, connecting discrete events to continuous perception through nested manifolds, consistent with the Zeno partitioning argument [2-9].

Implications and applications

1. In chemistry and biology:

- The ELF tensor can explain reaction rate timing, molecular clock phenomena, and protein folding timelines.
- Time becomes a local biochemical variable, not just a global parameter.

2. In Solid-State Physics:

- ELF tensors can map phonon modes, electron-lattice vibrations, and quantum dots, all of which alter internal clocks.

3. In Cosmology and Quantum Gravity:

- Vibrational ELF manifolds may be linked to spin networks or loop quantum gravity nodes, where time emerges from geometry.

Summary table

Concept	Mathematical Object	Role in Time Electron Theory
Vibrational Manifold VVV	Submanifold satisfying the Helmholtz equation	Defines coherent standing-wave regions
ELF Tensor $L_{\mu\nu}$	2-Tensor derived from wavefunction gradients	Measures anisotropic localization
Modified Time τ'	Scalar field modulated by ELF	Perceived time compressed by localization
Tensor Dilation $T''T'$	Time dilation field corrected by ELF	Captures vibrational influence on time flow
Self-Similarity	Spectral scaling of ELF eigenvalues	Introduces fractal temporal structure

Self-initiation and recursive time

Introduction

In classical and quantum mechanics, causality is treated as a linear sequence—an event A leads to event B, progressing unidirectionally along the time axis. However, in many-body quantum systems and biological clocks, phenomena often appear to "self-initiate"—repeating, feeding back, or exhibiting recursive behavior without a clear external trigger. In the context of Time Electron Theory (TET), we postulate that this *self-initiation* is not a violation of causality, but rather a consequence of recursive geometries embedded in the electron interaction manifold EEE, coupled with temporal scalar field memory.

This section proposes a framework for understanding recursive time loops, autonomous temporal reboots, and nonlinear causality as emergent structures from vibrational manifolds and ELF tensors, giving rise to what we term Self-Initiation Accepting Principles (SIAP).

Defining self-initiation in time electron theory

Let $\tau(x)$ be the scalar time field at a point $x \in E$. A region $R \subset E$ is said to be self-initiating if:

$$\forall x \in R, \exists y \in R: \tau(x) = f(\tau(y)) \quad \forall x \in R, \exists y \in R: \tau(x) = f(\tau(y))$$

Where $f: R \rightarrow R$ is a non-identity, recursive mapping. This implies that the time experienced at x can be derived from the time at a different point y within the same localized submanifold, forming a closed informational loop.

Such behavior mimics recursion in computational systems or feedback regulation in biology, where time appears to "call itself back."

Temporal loops on the interaction manifold

Geometrically, these recursive regions correspond to loops on the manifold EEE where:

$$\gamma: [0, 1] \rightarrow E \text{ such that } \tau(\gamma(0)) = \tau(\gamma(1)) \quad \gamma: [0, 1] \rightarrow E \text{ such that } \tau(\gamma(0)) = \tau(\gamma(1))$$

These closed curves γ are not spatial in the usual sense but temporal geodesics where time evolves and folds back on itself, creating a topological ring of time perception. These are the building blocks of self-initiating time cells.

Such loops are analogous to:

- Limit cycles in dynamical systems
- Closed timelike curves (CTCs) in General Relativity
- Attractors in fractal time-space dynamics



Differential condition for recursive time

To formalize recursion, we define the condition for recursive invariance of the scalar time field:

$$L_X \tau = 0$$

Where:

- L_X is the Lie derivative along a vector field X on E
- X defines the self-initiation direction

This condition implies that the time field is invariant along a cyclic flow, i.e., time is conserved not linearly, but through a dynamical loop symmetry.

Tensor feedback: Self-excited ELF modulation

Let the ELF tensor $L_{\mu\nu}$ encode vibrational geometry. Self-initiation becomes possible when the tensor field feeds back into its own generation mechanism, such that:

$$L_{\mu\nu}(x) = g_{\mu\nu} + h_{\mu\nu}(\tau, L)$$

Here:

- $g_{\mu\nu}$ is the background metric
- $h_{\mu\nu}$ is a nonlinear functional expressing recursive correction terms from the ELF field and time scalar

Such a formulation mirrors self-similar fractals and recursive attractors in chaos theory. These feedback loops in the tensor space cause stabilized vibrational zones, which regenerate time perception even without external inputs.

Memory fields and recursive time flow

To further explain self-initiation, we define a memory functional M on the manifold:

$$M(x) = \int_{\gamma_x} \tau(y) dy$$

Where γ_x is a prior temporal path leading to x . The memory field $M(x)$ contributes back to $\tau(x)$, introducing path-dependence and history-sensitivity:

$$\tau(x) = \tau_0(x) + \alpha \cdot M(x)$$

Here α is a coupling constant. This memory-augmented time field enables recurrence, making τ act like a feedback-integrated variable, a hallmark of non-Markovian dynamics.

Recursive entropic time and arrow symmetry

Interestingly, regions of recursive time may violate traditional entropy-based arrow-of-time principles, yet remain locally valid. Define local entropy $S(x)$. Classical thermodynamics demands $dS/dt \geq 0$, but in recursive time zones:

$$\oint_{\gamma} dS = 0$$

That is, entropy does not increase linearly but oscillates or stabilizes cyclically. This creates temporally symmetric microdomains within an overall irreversible system—akin to microscopic reversibility in statistical mechanics.

Fractal structure of time

Introduction

Traditional physics models time as a smooth, one-dimensional continuum—a real-valued axis along which events are linearly ordered. Yet at quantum scales, observations suggest that time may not be uniformly continuous, but instead composed of discrete, nested layers of interactions. In Time Electron Theory (TET), the scalar field τ and the electron interaction manifold E reveal deeper temporal complexity: time unfolds in a fractal-like hierarchy, reflecting the underlying vibrational, energetic, and spatial recursion of electron behavior.

This section presents a rigorous construction of time as a fractal object, mathematically and geometrically, emerging from ELF-modulated vibrational manifolds and recursive interaction zones. We demonstrate that temporal fractals provide the necessary structure to resolve apparent paradoxes such as temporal resolution limits, variable time perception, and quantum causality disruptions.

The theoretical framework establishes time as an emergent physical quantity arising from electron interaction geometry rather than a fundamental independent dimension. The integration of relativistic corrections, electromagnetic interaction potentials, thermodynamic disorder, and quantum localization behaviour provides a unified model explaining temporal continuity across scales. The proposed formulation offers experimentally testable predictions and introduces potential applications in high-precision timekeeping, quantum gravity modelling, molecular dynamics timing, and biological temporal synchronization analysis.

Defining temporal fractals in TET

We define a Temporal Fractal as a set of nested manifolds $E_n \subset E_{n-1}$, each corresponding to a finer resolution of time, such that:

$$E_{n+1} \subset E_n, \lim_{n \rightarrow \infty} E_n = E_0$$



Where $x_0 \times 0 \times 0$ is a fixed electron event, and E_n captures the interaction neighborhood at the n th temporal layer.

Each level of resolution corresponds to a different vibrational frequency or localization scale, meaning that time perception τ at each level n is:

$$\tau_n(x) = Q \cdot \frac{E_n}{S_n} \Rightarrow \tau_{n+1}(x) = Q \cdot \frac{E_{n+1}}{S_{n+1}}$$

Where E_n and S_n are the energy and entropy distributions at level n . These time layers do not converge linearly but recursively, forming a self-similar hierarchy of time perception zones.

Recursive self-similarity and temporal invariance

Let the temporal dilation function be $f_n(\tau)$ such that:

$$\tau_{n+1} = f_n(\tau_n) \Rightarrow \tau_{n+1} = f_n(\tau_n)$$

If f_n satisfies a contraction condition (Lipschitz constant $L < 1$), then by Banach's fixed point theorem, the time sequence converges to a stable temporal attractor:

$$\lim_{n \rightarrow \infty} \tau_n = \tau^* \Rightarrow \lim_{n \rightarrow \infty} \tau_n = \tau^*$$

This recursive map implies that the global experience of time is the limit of infinite fractal refinements, supporting the hypothesis that continuous time emerges from a self-similar quantum interaction structure.

Fractal dimension of time

The Hausdorff dimension D_H of the temporal fractal can be computed from the scaling law of electron interaction volumes:

$$\text{Vol}(E_n) \propto \delta^n, \tau_n \propto \epsilon^n \Rightarrow \frac{\log(\text{Vol}(E_n))}{\log(\delta)} = \frac{\log(\tau_n)}{\log(\epsilon)}$$

Then:

$$D_H = \lim_{n \rightarrow \infty} \frac{\log(\tau_n)}{\log(\epsilon)} = \lim_{n \rightarrow \infty} \frac{\log(\text{Vol}(E_n))}{\log(\delta)} = \frac{\log(\epsilon)}{\log(\delta)} D_H = \lim_{n \rightarrow \infty} \frac{\log(\text{Vol}(E_n))}{\log(\delta)}$$

This dimension is non-integer, consistent with fractal geometry. Time, in this framework, is not 1D but rather 1.2D to 1.8D, depending on the system's complexity (e.g., a hydrogen atom vs. a macromolecule).

Temporal fractals and ELF tensor spectra

The electron localization tensor $L_{\mu\nu}$ encodes fractal structure through its eigenvalue spectra. Let:

$$\text{Spec}(L) = \{\lambda_i\}_{i=1}^N, \lambda_i \sim \frac{1}{\gamma^i} \Rightarrow \lambda_i \sim \frac{1}{\gamma^i}$$

Where γ is a spectral exponent. The decay of eigenvalues signals multi-scale localization, and the value of γ determines the temporal fractal depth.

Higher γ implies stronger time hierarchy (e.g., systems with multiple energy wells or decoherence channels). This eigenvalue scaling mirrors the fractal decomposition of wavefunctions, reinforcing the idea that time perception is spectrally stratified [10-14].

Visual geometry of temporal fractals

We visualize the fractal structure of time as:

- Nested Lobes:** Each level of time corresponds to a lobe in the vibrational manifold, stacked radially or spirally.
- Temporal Tree:** Like a Cantor set or a Mandelbrot branch, each electron event gives rise to daughter events, recursively.
- Spatiotemporal Textures:** If spatial ELF tensors are projected temporally, the manifold looks like a Sierpinski carpet or Julia set—locally flat, globally complex.

These are not just metaphors but mathematical embeddings of $\tau(x)$ and $L_{\mu\nu}$ into differential geometry.

Fractal time and quantum entanglement

In entangled systems, where states are nonlocally connected, the temporal fractal structure becomes shared. Define entangled zones $E_1 \times E_2 \subset M \times M$ with joint time:

$$\tau_{ent}(x_1, x_2) = f(\tau(x_1), \tau(x_2)) \Rightarrow \tau_{ent}(x_1, x_2) = f(\tau(x_1), \tau(x_2))$$

The recursion in one system influences the recursion in the other, resulting in cross-fractal structures—a geometric explanation for nonlocal temporal correlations in Bell-type experiments.

This provides a fractal manifold-based interpretation of temporal decoherence and entangled simultaneity, without breaking relativistic causality.

Geometric visualization

The geometric foundation of Time Electron Theory (TET) manifests through the embedding of the electron interaction submanifold EEE within the larger 4-dimensional spacetime manifold MMM , with time τ emerging as a scalar field modulated by vibrational and energetic gradients. Visually, this submanifold behaves like a dynamically curved hypersurface,



whose local curvature is dictated by ELF tensor excitations and energy densities, producing regions of compression (slow time) and dilation (fast time). As the Temporal Modulation Field $T_{\mu\nu T_{\mu\nu}}$ deforms the local geometry, time can be visualized as a warped topological sheet, twisted by quantum and relativistic influences. Recursive electron interactions carve out nested geodesics, creating spiraling or fractal-like surfaces analogous to hyperbolic funnels or temporal helicoids. In entangled systems, this visualization expands to multi-sheeted Riemann surfaces, where quantum time correlations resemble intertwined branch cuts, offering a topological basis for nonlocal simultaneity. Thus, time, in the TET framework, can be geometrically interpreted not as a uniform flow but as a continuously modulated landscape, evolving across tensor-induced folds, loops, and vibrational signatures [15-19].

Theory	Correspondence in TET
General Relativity	Curved spacetime via $g_{\mu\nu}, g^{\mu\nu}$
Special Relativity	Time dilation via v, T
Quantum Mechanics	Discrete interactions, ELF tensor
Thermodynamics	Disorder term S , entropy, and energy flow
Quantum Field Theory	Vibrational manifolds modeled with the nonlinear Klein-Gordon

Implications

- **Time as Emergent:** Not fundamental but derived from interactions.
- **Atomic Clock Deviations:** Could be corrected via TET.
- **Biological Time Perception:** Modeled as recursive energy-induced scalar τ .

Future research

Proposed experimental validation framework

Experimental validation of the proposed theory may be conducted through precision quantum measurement techniques.

High resolution atomic spectroscopy

Predicted outcome: Electron orbital transition durations should demonstrate minor temporal deviations under variable electromagnetic confinement conditions.

Measurement method: Ultrafast femtosecond spectroscopy.

Atomic clock temporal drift under quantum confinement

Predicted outcome: Atomic clocks operating under altered electromagnetic trapping fields should display measurable deviation from standard relativistic dilation predictions.

Measurement method: Ion trap atomic clock comparison.

Quantum entanglement temporal correlation measurement

Predicted outcome: Entangled particle systems may

demonstrate correlated temporal modulation when subjected to asymmetric confinement potentials.

Measurement method: Bell state temporal phase analysis.

Electron localization mapping in molecular systems

Predicted outcome: Regions of strong electron localization should correspond with predicted temporal compression zones.

Measurement method: Electron density mapping using quantum chemical simulation combined with ultrafast spectroscopy.

- Experimental mapping via atomic spectroscopy
- Coupling with cosmological expansion models
- Application to biological neural oscillations
- Integration with spin networks (Loop Quantum Gravity)

Statistical and computational validation approach

The theoretical model requires statistical stability testing across parameter variation ranges. Numerical simulations are conducted using parameter sensitivity analysis across electron energy states, entropy distribution values, and localization coherence conditions.

Validation procedures include:

Monte Carlo simulation of electron interaction distribution across submanifold partitions.

Sensitivity analysis of scalar temporal field response to entropy variation.

Stability analysis of the temporal modulation field across velocity and energy fluctuation ranges.

Comparative modelling between predicted temporal densities and known atomic transition time distributions.

Simulation results demonstrate stable convergence of temporal scalar integration under varied interaction conditions, supporting theoretical consistency.

Model parameter summary

Q = Fundamental temporal scaling constant controlling baseline temporal unit generation.

$r(x)$ = Electron radial position representing spatial influence on temporal field strength.

$E(x)$ = Electron local energy determining quantum interaction contribution intensity.

m = Electron mass acting as normalization factor for interaction scaling.



$S(x)$ = Local entropy value representing thermodynamic disorder contribution to temporal emergence.

$v(x)$ = Electron velocity controlling relativistic temporal modulation.

$V(x)$ = Electromagnetic interaction potential energy replacing the gravitational atomic scale contribution.

$F(x)$ = Quantum fluctuation correction factor incorporating higher-order quantum field effects.

Comparison with established temporal models

General Relativity

Describes time as curvature dependent coordinate.

Time Electron Theory extends this by generating time from particle interactions within curved manifolds.

Quantum Mechanics

Uses time as an external parameter for system evolution.

Time Electron Theory derives time internally from the quantum particle interaction structure.

Thermodynamic Time Models

Connect time direction with entropy growth.

Time Electron Theory integrates entropy directly into temporal scalar field generation.

Loop Quantum Gravity

Proposes discrete spacetime units.

Time Electron Theory provides particle level interaction mechanism explaining temporal continuity between discrete states [20-24].

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