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Research Article

About One Generalization of the Classical Physics Problem of Controlled Inverted Pendulum

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Abstract

A problem of a couple of connected inverted pendulums is considered under stochastic perturbations of the type of the Wiener and Poisson processes. This problem generalizes the classical physics problem of a controlled inverted pendulum and continues a series of unsolved problems in stability and optimal control theory for stochastic delay differential equations, published by the author from 2010 to the present. Since none of these problems has been solved yet, the purpose of this paper is to attract the attention of interested researchers to these problems, which may lead both to the solution of at least some of them and to the general development of stability and optimal control theory for stochastic systems.

More Information

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Introduction

The classical physics problem of a controlled inverted pendulum has a long history [1] and, to date, remains very popular in research (see, for instance [2-7] and the references therein).

Consider the system of two differential equations:

$$\ddot{x}_1(t)-a_1\sin x_1(t)-b_1x_2(t)=u_1(t), \ddot{x}_2(t)-a_2\sin x_2(t)-b_2x_1(t)=u_2(t),\ a_1>0,\ a_2>0,\ t\geq 0, \tag{1}$$

with the controls

$$u_1(t)=\int_0^\infty dK_1(\tau)x_1(t-\tau),\quad u_2(t)=\int_0^\infty dK_2(\tau)x_2(t-\tau). \tag{2}$$

If $b_1=b_2=0$, then the system (1), (2) splits into two independent equations, each of which describes a problem of a controlled inverted pendulum. So, the system (1) can be considered as a system of two coupled inverted pendulums [8].

The classical way of stabilisation in this problem [1] is a linear combination of the state $x(t)$ and the velocity $\ddot{x}(t)$ of the pendulum, i.e.,

$$u(t)=\alpha x(t)+\beta\dot{x}(t).$$

But this type of control, which provides instantaneous feedback, is quite difficult to realise because it usually requires a finite time to take measurements of the coordinates and velocities, process the results, and implement them in the control action.

Unlike the classical approach to stabilisation, another stabilisation method is considered in [9]. It is supposed that only the trajectory of the controlled inverted pendulum is observed and the control $u(t)$ does not depend on the velocity, but depends on the all previous values of the trajectory $x(s)$, $s\leq t$, and is given in the form (2), where the kernel $K_i(\tau)$, $i=1,2$, is a continuous from the right function of bounded variation on $[0,\infty)$. The integral is understood in the sense of Stieltjes. It means, in particular, that both distributed and discrete delays can be used depending on the concrete choice of the kernel $K_i(\tau)$.

Stochastic perturbations and equilibria

Let $\{\Omega, \mathfrak{F}, \mathbf{P}\}$ be a complete probability space, $\{\mathfrak{F}_t, t \geq 0\}$ be a nondecreasing family of sub-σ-algebras of $\mathfrak{F}$, i.e., $\mathfrak{F}_s \subset \mathfrak{F}_t$ for $s < t$, and **E** be the mathematical expectation with respect to the probability **P**.

Supposing that stochastic perturbations influence the parameters $a1$ and $a2$ in the system (1)

$$a_i \to a_i + \dot{\xi}_i(t), \quad \xi_i(t) = \sigma_i w_i(t) + \gamma_i \tilde{\nu}_i(t), \quad i = 1,2,$$

where $\tilde{\nu}_i(t) = \nu_i(t) - \lambda_i t$, $\mathbf{E}\nu_i(t) = \lambda_i t$, $\lambda_i > 0$, $w_i(t)$ and $\nu_i(t)$ are, respectively, $\mathfrak{F}_t$ -measurable mutually independent of the Wiener and Poisson processes, we obtain the system of stochastic differential equations with delays [9-11].

$$\dot{x}_{11}(t) = x_{12}(t), \dot{x}_{12}(t) = (a_1 + \dot{\xi}_1(t))\sin x_{11}(t) + b_1 x_{21}(t) + \int_0^\infty dK_1(\tau) x_{11}(t-\tau), \dot{x}_{21}(t) = x_{22}(t), \dot{x}_{22}(t) = (a_2 + \dot{\xi}_2(t))\sin x_{21}(t) + b_2 x_{11}(t) + \int_0^\infty dK_2(\tau) x_{21}(t-\tau).$$

or

$$dx_{11}(t) = x_{12}(t)dt, dx_{12}(t) = \left(a_1 \sin x_{11}(t) + b_1 x_{21}(t) + \int_0^\infty dK_1(\tau) x_{11}(t-\tau)\right)dt + \sin x_{11}(t)(\sigma_1 dw_1(t) + \gamma_1 d\tilde{\nu}_1(t)), dx_{21}(t) = x_{22}(t)dt, dx_{22}(t) = \left(a_2 \sin x_{21}(t) + b_2 x_{11}(t) + \int_0^\infty dK_2(\tau) x_{21}(t-\tau)\right)dt + \sin x_{21}(t)(\sigma_2 dw_2(t) + \gamma_2 d\tilde{\nu}_2(t)). \quad (3)$$

Note that the system (1), (2) has both the zero equilibrium and the nonzero equilibrium $(\hat{x}_1, \hat{x}_2)$, which is a solution of the system of algebraic equations

$$a_1 \sin\hat{x}_1 + b_1 \hat{x}_2 + k_{01}\hat{x}_1 = 0, a_2 \sin\hat{x}_2 + b_2 \hat{x}_1 + k_{02}\hat{x}_2 = 0,$$

Where $k_{0i} = \int_0^\infty dK_i(\tau)$, $i = 1,2$, and is also the equilibrium of the system of stochastic differential equations

$$dx_{11}(t) = x_{12}(t)dt, dx_{12}(t) = \left(a_1 \sin x_{11}(t) + b_1 x_{21}(t) + \int_0^\infty dK_1(\tau) x_{11}(t-\tau)\right)dt + (x_{11}(t) - \hat{x}_1)(\sigma_1 dw_1(t) + \gamma_1 d\tilde{\nu}_1(t)), dx_{21}(t) = x_{22}(t)dt, dx_{22}(t) = \left(a_2 \sin x_{21}(t) + b_2 x_{11}(t) + \int_0^\infty dK_2(\tau) x_{21}(t-\tau)\right)dt + (x_{21}(t) - \hat{x}_2)(\sigma_2 dw_2(t) + \gamma_2 d\tilde{\nu}_2(t)). \quad (4)$$

The problems of stability investigation of the systems (3) and (4) are now unsolved problems and are proposed to the readers' attention.

Conclusion

The proposed paper continues a series of papers devoted to unsolved problems in the theory of stability and optimal control for stochastic systems (see [12-26] and the references therein). Here, the problem of the controlled inverted pendulum is considered, which was first studied many years ago [1] and remains very popular in research (see, for instance, [2-7] and the references therein). In particular, it is supposed that the inverted pendulum is considered under the influence of stochastic perturbations, so the considered stabilisation problem is a problem of the theory of stochastic functional differential equations [9-11].

The list of the author's unsolved problems continues to increase. It is hoped that these unsolved problems, when collected together, will enable interested researchers to identify fundamentally new ideas for their solution. An excellent outcome of studying the proposed unsolved problems here would be the publication of a series of new papers with their solutions.

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