

## Research Article

# Local Representability in Finite-Capacity Causal Horizons: A Phenomenological Approach to Effective Cosmology a Testable Framework for Holographic Saturation The Effective 17% Density Excess, and the Geometric-Informational Partition

**Cristian Antiba Carvajal Bertolini\***

GEII-FCEIA-UNR, Rosario, Argentina

## More Information

**\*Corresponding author:** Cristian Antiba Carvajal Bertolini, GEII-FCEIA-UNR, Rosario, Argentina, Email: antiba@fceia.unr.edu.ar

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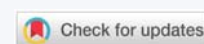
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## Abstract

The holographic principle suggests that the physically accessible information contained inside a causal region may ultimately be constrained by the entropy associated with its boundary. Motivated by this idea, we investigate a phenomenological framework in which causal horizons possess a finite effective information capacity that limits the local semiclassical representability of bulk degrees of freedom while preserving global unitary evolution.

The proposed framework does not modify microscopic quantum dynamics. Instead, it assumes that as a causal horizon approaches an effective saturation regime, the reconstruction of bulk physics by a local semiclassical observer gradually becomes incomplete, whereas the complete physical state remains encoded globally through nonlocal quantum correlations.

An effective entropy-area response density is introduced as a phenomenological quantity describing the local informational cost associated with representing bulk excitations on a causal boundary. A complementary geometric-informational partition is then postulated as a normalization hypothesis motivated by spherical geometry.

The framework is further compared with the observational discrepancy between early- and late-time determinations of the Hubble parameter, expressed as an effective density excess of approximately 17%. This observational quantity is not identified with the geometric normalization itself, but rather provides a phenomenological scale against which the proposed mechanism may eventually be tested.

The present work therefore introduces a falsifiable theoretical program rather than a complete cosmological model. Its principal objective is to establish a mathematically consistent framework from which microscopic horizon dynamics may later derive quantitative corrections to cosmological evolution.



## Introduction

Modern gravitational physics increasingly suggests that spacetime geometry, thermodynamics, and quantum information are deeply interconnected.

Beginning with the work of Bekenstein and Hawking, black-hole entropy established that the number of physically distinguishable states associated with a gravitating system scales with the area of its boundary rather than with its enclosed volume. Subsequent developments, including the holographic principle, entanglement entropy, modular Hamiltonians, and emergent gravity programs, indicate that spacetime geometry may represent an effective macroscopic manifestation of microscopic quantum-information dynamics rather than an entirely fundamental entity.

These ideas motivate the possibility that causal horizons possess a finite operational capacity for encoding locally reconstructible physical information. Such a limitation would not imply any loss of microscopic information or violation of quantum unitarity, but rather a restriction on the amount of information that admits a complete semiclassical description inside a given causal region. This distinction between global microscopic evolution and local semiclassical representability constitutes the central conceptual assumption of the present work.

Unlike conventional approaches to the Hubble tension, which generally introduce additional dynamical fields or modifications of cosmological evolution, the present proposal investigates whether part of the observed discrepancy could instead emerge from finite-capacity limitations associated with local semiclassical reconstruction.

No claim is made that the mechanism presented here already constitutes a complete explanation of cosmological observations. Instead, the objective is to formulate a mathematically explicit framework capable of generating quantitative predictions that may eventually be tested against independent observational constraints.

## Foundational hypotheses and scope

To distinguish clearly between assumptions, derived results and physical interpretation, the present work is organized around the following foundational hypotheses.

### Hypothesis I: Preservation of microscopic unitarity

The microscopic quantum evolution of the Universe is assumed to remain exactly unitary. No mechanism proposed in this work destroys information or modifies quantum mechanics.

### Hypothesis II: Finite operational capacity of causal horizons

Motivated by the Bekenstein-Hawking entropy law, causal

horizons are assumed to possess a finite effective capacity for representing distinguishable physical configurations. This hypothesis should not be interpreted as a theorem derived from the entropy-area law itself, but as a phenomenological extension inspired by holographic arguments.

### Hypothesis III: Local semiclassical representability

The observable semiclassical description accessible to local observers is assumed to require finite informational resources. When the effective capacity of a causal horizon approaches saturation, part of the physical information may cease to admit a complete local semiclassical representation while remaining globally encoded within the quantum state.

### Hypothesis IV: Effective cosmological manifestation

If finite-capacity effects become relevant on cosmological scales, they may contribute an effective correction to macroscopic gravitational dynamics. The present article does not derive such corrections from first principles, but proposes a mathematical framework intended to guide future microscopic derivations.

### Scope of the present work

Throughout this manuscript, we distinguish carefully between

1. explicit mathematical definitions,
2. phenomenological hypotheses,
3. quantities derived within the proposed framework,
4. speculative physical interpretation.

Accordingly, every equation introduced below should be interpreted according to its corresponding logical status. The principal contribution of the present work is therefore the formulation of a coherent and internally consistent phenomenological framework whose validity ultimately depends upon future theoretical derivations and experimental confrontation.

## Theoretical background

The proposed framework combines concepts originating from several independent research programs that have emerged during the last few decades.

First, black-hole thermodynamics establishes that gravitational systems possess entropy proportional to horizon area.

Second, quantum-information theory demonstrates that entanglement entropy provides a quantitative description of information shared between causally separated subsystems.

Third, modular Hamiltonians relate variations of entanglement entropy to local energy perturbations through the first law of entanglement.

Finally, several approaches to emergent gravity suggest that spacetime dynamics may arise from coarse-grained microscopic degrees of freedom.

The present proposal does not attempt to derive these theories from one another, but investigates whether they may consistently support a common phenomenological description based on finite local representability.

## Finite holographic capacity

One of the central motivations of the present framework is the entropy–area relation established by black-hole thermodynamics,

$$S_{\text{BH}} = \frac{k_B A}{4\ell_P^2}, \quad (1)$$

where

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad (2)$$

Denotes the Planck length.

Equation (1) establishes the maximum entropy associated with a gravitational horizon and suggests that the physically distinguishable degrees of freedom of a bounded region may scale with boundary area rather than with enclosed volume. The present work adopts this result only as physical motivation: no claim is made that Eq. (1) directly implies the existence of a local capacity bound on arbitrary quantum states.

Instead, we introduce the following phenomenological postulate.

### Definition 1. Effective capacity

Let  $\mathcal{H}_R$  denote the Hilbert space associated with a causal region  $R$ . We define the effective representational capacity as the maximum amount of locally reconstructible information that admits a semiclassical description inside that region. This quantity should be understood as an operational concept rather than as a microscopic counting theorem.

### Definition 2. capacity saturation

A causal horizon is said to approach saturation whenever additional physical excitations no longer admit independent local semiclassical reconstruction without increasing coarse graining. Importantly, *capacity saturation does not imply information loss*. Instead, it represents the gradual transition between complete local reconstruction and globally encoded quantum information.

## Entanglement entropy and modular response

Consider a causally complete region  $R$  described by the reduced density matrix

$$\rho_R. \quad (3)$$

Its entanglement entropy is

$$S_R = -\text{Tr}(\rho_R \ln \rho_R). \quad (4)$$

The corresponding modular Hamiltonian satisfies

$$\rho_R = \frac{e^{-K_R}}{\text{Tr}(e^{-K_R})}. \quad (5)$$

For perturbations around a reference state,

$$\delta S_R = \delta \langle K_R \rangle, \quad (6)$$

This constitutes the first law of entanglement. Equation (6) establishes a direct relationship between changes in entropy and changes in modular energy and motivates the introduction of an effective entropy–area response operator.

## Effective entropy–area response

We define the effective entropy–area response density by

$$\delta S_R = \int_{\partial R} \Sigma(x) dA. \quad (7)$$

Accordingly,

$$\Sigma(x) \equiv \frac{\delta S_R}{\delta A(x)} \quad (8)$$

measures the infinitesimal variation of entanglement entropy associated with a local deformation of the causal boundary.

Unlike the Bekenstein–Hawking entropy itself,  $\Sigma(x)$  is not introduced as a universal quantity, but as an effective phenomenological response function characterizing the local informational cost required to encode bulk excitations.

### Capacity criterion

Motivated by Eq. (1), we introduce the phenomenological inequality

$$\Sigma(x) \leq \Sigma_{\text{crit}} \quad (9)$$

with

$$\Sigma_{\text{crit}} = \frac{1}{4\ell_P^2}. \quad (10)$$

Equation (9) defines the effective saturation criterion adopted throughout the remainder of this work and should not be interpreted as a theorem.

## Macroscopic horizon dynamics

The microscopic evolution responsible for redistributing entanglement across the causal boundary remains unknown. Consequently, rather than proposing a fundamental dynamical equation, we introduce the following phenomenological transport model,

$$\frac{\partial \Sigma}{\partial t} = D_H \nabla_H^2 \Sigma + J_{\text{bulk}}, \quad (11)$$

where

- $D_{H_-}$  denotes an effective horizon transport coefficient,
- $\nabla_H^2$  is the Laplace-Beltrami operator intrinsic to the causal boundary,
- $J_{bulk}$  represents the injection of entanglement generated by bulk excitations.

Equation (11) should be regarded as an effective coarse-grained description rather than a microscopic law.

A corresponding quasi-local response kernel may be parameterized as

$$\Sigma(x, t) = \Sigma_0 \exp \left[ -\frac{|x - x_H|^2}{2\lambda_H^2} \right] V(t), \quad (12)$$

where  $\lambda_H$  defines an effective coarse-graining length. Different microscopic theories may produce different kernels; Eq. (12) merely illustrates one possible phenomenological realization of finite-capacity redistribution.

## Transition of local representability

The effective response operator naturally suggests three qualitatively distinct physical regimes,

$$\Sigma(x) \leq \Sigma_{crit} \text{ complete local semiclassical reconstruction} \quad (13)$$

$$\Sigma \rightarrow \Sigma_{crit} \Rightarrow \text{progressive loss of local representability}, \quad (14)$$

$$\Sigma \simeq \Sigma_{crit} \Rightarrow \text{globally preserved but locally incomplete encoding} \quad (15)$$

The transition proposed here therefore concerns only the operational accessibility of information: the underlying quantum state is assumed to remain perfectly unitary throughout the entire evolution.

## Geometric-informational partition

The phenomenological framework introduced in the preceding sections requires an effective normalization relating the maximum locally representable sector to its complementary geometrical contribution.

Rather than introducing an arbitrary normalization constant, we seek a dimensionless quantity naturally associated with the geometry of approximately spherical causal horizons. For a sphere of radius  $R_H$  and diameter

$$D_H = 2R_H, \quad (16)$$

Its volume may be written as

$$V_H = \frac{\pi}{6} D_H^3. \quad (17)$$

Equation (17) contains the dimensionless coefficient

$$\frac{\pi}{6} = 0.5235987756... \quad (18)$$

Which is purely geometric. No physical interpretation

follows directly from Eq. (18); geometry supplies only the dimensionless coefficient  $\pi/6$ . The present work introduces the following independent phenomenological hypothesis.

## Closure hypothesis

We define

$$D_{rep} \equiv \frac{\Sigma_{rep}}{\Sigma_{crit}}, \quad (19)$$

where

$$0 \leq D_{rep} \leq 1. \quad (20)$$

Its complementary contribution is

$$D_{grav} = 1 - D_{rep}. \quad (21)$$

The central normalization hypothesis proposed in this work is

$$D_{rep} = \sqrt{\frac{\pi}{6}} \quad (22)$$

which immediately gives

$$D_{grav} = 1 - \sqrt{\frac{\pi}{6}} \quad (23)$$

with the numerical values

$$D_{rep} = 0.723601255, \quad (24)$$

$$D_{grav} = 0.276398745. \quad (25)$$

The square-root normalization is introduced as an independent physical postulate motivated by representability arguments; it is not a mathematical consequence of Euclidean geometry. Consequently,

$$D_{rep} + D_{grav} = 1 \quad (26)$$

defines an effective partition between locally reconstructible information and its complementary geometrical contribution.

## Interpretation

Within the present framework,

- $D_{rep}$  measures the maximum fraction of the microscopic information that remains accessible through local semiclassical reconstruction.
- $D_{grav}$  measures the complementary fraction whose influence survives only through effective macroscopic geometry.

Neither quantity should presently be interpreted as directly observable. Instead, they represent internal parameters of the phenomenological model whose validity must ultimately be established through independent cosmological tests.

## Observed effective density excess

The current observational discrepancy between early-



and late-Universe determinations of the Hubble constant motivates the introduction of an effective density parameter.

Let

$$H_0^{\text{early}} \tag{27}$$

denote the value inferred from early-Universe observations, and

$$H_0^{\text{late}} \tag{28}$$

The corresponding local determination. We define

$$\delta_H = \frac{H_0^{\text{late}} - H_0^{\text{early}}}{H_0^{\text{early}}}. \tag{29}$$

Using the Friedmann equation,

$$H^2 = \frac{8\pi G}{3} \rho, \tag{30}$$

The corresponding effective density excess becomes

$$\sigma_{\text{obs}} = \left( \frac{H_0^{\text{late}}}{H_0^{\text{early}}} \right)^2 - 1 \tag{31}$$

Using representative observational values,

$$H_0^{\text{early}} = 67.4 \text{ kms}^{-1} \text{ Mpc}^{-1}, \tag{32}$$

$$H_0^{\text{late}} = 73.0 \text{ kms}^{-1} \text{ Mpc}^{-1}, \tag{33}$$

one obtains

$$\sigma_{\text{obs}} = 0.173 \pm 0.01, \tag{34}$$

corresponding to an effective density difference of approximately

$$17.3\% \tag{35}$$

### Conceptual separation

One of the principal conceptual clarifications introduced in this revised manuscript is the explicit distinction among three independent numerical quantities,

$$\frac{\pi}{6} = 0.5236, \tag{36}$$

$$D_{\text{grav}} = 0.2764, \tag{37}$$

$$\sigma_{\text{obs}} \approx 0.173. \tag{38}$$

These numbers possess completely different physical meanings. The coefficient  $\pi/6$  originates from Euclidean geometry. The quantity  $D_{\text{grav}}$  belongs exclusively to the proposed normalization hypothesis. Finally,  $\sigma_{\text{obs}}$  is an empirical quantity inferred from cosmological observations. Any future microscopic theory reproducing  $\sigma_{\text{obs}}$  must therefore explain why the observational value remains below the theoretical upper amplitude represented by  $D_{\text{grav}}$ .

### Two phenomenological saturation models

The present framework admits two natural saturation parameterizations. The first ignores representability

weighting. The second explicitly incorporates the geometric-informational partition. Both models are retained because they correspond to different physical assumptions.

#### Model A: pure area saturation

We first assume

$$\sigma_{\text{sat}} = \left( \frac{R_H}{R_{\text{lim}}} \right)^2. \tag{39}$$

Equating

$$\sigma_{\text{sat}} = \sigma_{\text{obs}}, \tag{40}$$

gives

$$R_{\text{lim}}^{(\text{A})} = \frac{R_H}{\sqrt{\sigma_{\text{obs}}}}. \tag{41}$$

Using

$$R_H = 14.4 \text{ Gpc}, \tag{42}$$

one obtains

$$R_{\text{lim}}^{(\text{A})} = 34.6 \text{ Gpc}. \tag{43}$$

#### Model B: representability-weighted saturation

Including the complementary representability factor,

$$\sigma_{\text{sat}} = D_{\text{grav}} \left( \frac{R_H}{R_{\text{lim}}} \right)^2. \tag{44}$$

Hence,

$$R_{\text{lim}}^{(\text{B})} = R_H \sqrt{\frac{D_{\text{grav}}}{\sigma_{\text{obs}}}}. \tag{45}$$

The numerical result becomes

$$R_{\text{lim}}^{(\text{B})} = 18.2 \text{ Gpc}. \tag{46}$$

The smaller limiting radius reflects the fact that only the complementary representability sector contributes to the effective correction (Table 1).

### Effective cosmological correction

The preceding sections establish a phenomenological framework in which finite horizon capacity may limit local semiclassical representability. The remaining question is how such microscopic limitations could manifest themselves within large-scale cosmological dynamics.

At the present stage, no first-principles derivation exists. Accordingly, the following equations should be interpreted as an effective parameterization rather than as a derived theory.

| Table 1: Comparison of the two phenomenological saturation models. |                 |                                            |
|--------------------------------------------------------------------|-----------------|--------------------------------------------|
| Assumption                                                         | Limiting Radius | Interpretation                             |
| Pure area aturation                                                | 34.6 Gpc        | Entire correction attributed to saturation |
| Representability weighted                                          | 18.2 Gpc        | Only the complementary sector contributes  |





## General effective friedmann equation

Let

$$H_{\Lambda\text{CDM}}(z) \quad (47)$$

Denote the expansion rate predicted by the standard cosmological model. We define the effective expansion law

$$H_{\text{eff}}^2(z) = H_{\Lambda\text{CDM}}^2(z)[1 + \Phi(z)] \quad (48)$$

where

$$\Phi = \Phi\left(\frac{\Sigma}{\Sigma_{\text{crit}}}, D_{\text{grav}}, \frac{R_H}{R_{\text{lim}}}, z\right) \quad (49)$$

is an unknown dimensionless response function. Its microscopic derivation constitutes the principal open problem of the present framework.

## Minimal parameterization

Although the complete functional form remains unknown, consistency requires several general properties.

First,

$$\Phi = 0 \quad (50)$$

whenever

$$\Sigma \ll \Sigma_{\text{crit}}, \quad (51)$$

Since finite-capacity effects should disappear far from saturation.

Second,

$$\Phi \geq 0 \quad (52)$$

because the proposed mechanism is intended to represent an additional effective contribution rather than a negative energy component.

A minimal phenomenological parameterization satisfying these conditions is

$$\Phi = D_{\text{grav}} \left(\frac{\Sigma}{\Sigma_{\text{crit}}}\right)^\alpha \left(\frac{R_H}{R_{\text{lim}}}\right)^\beta F(z), \quad (53)$$

where

- $\alpha > 0$  governs the sensitivity to local saturation,
- $\beta > 0$  controls the horizon-size dependence,
- $F(z)$  describes cosmological evolution.

Equation (53) should be regarded only as the simplest admissible parameterization. Different microscopic theories may predict different functional forms.

## Asymptotic behaviour

A physically acceptable function  $\Phi$  should satisfy

$$\Phi \rightarrow 0, R_H \ll R_{\text{lim}}, \quad (54)$$

$$\Phi \rightarrow D_{\text{grav}}, R_H \rightarrow R_{\text{lim}}, \quad (55)$$

$$\Phi < 1, \forall z. \quad (56)$$

These conditions guarantee that the correction remains finite and bounded.

## Quantitative predictions

Unlike many qualitative proposals, the present framework generates several quantitative predictions that may be independently tested. Agreement with only one observable would not be sufficient: the model should reproduce all observational constraints simultaneously.

### Prediction 1

If finite-capacity effects are physically real, the effective cosmological correction must remain bounded above by

$$\Phi_{\text{max}} = D_{\text{grav}} = 0.2764. \quad (57)$$

Consequently, future observations should never require

$$\Phi > 0.2764 \quad (58)$$

Within the validity domain of the model.

### Prediction 2

The effective correction should increase monotonically with

$$\left(\frac{\Sigma}{\Sigma_{\text{crit}}}\right). \quad (59)$$

Regions further from saturation should become observationally indistinguishable from standard  $\Lambda$ CDM.

### Prediction 3

If the proposed mechanism contributes to the Hubble tension, its amplitude should evolve smoothly with cosmic time. Therefore, future high-redshift determinations of the expansion history should constrain the function

$$F(z). \quad (60)$$

### Prediction 4

The framework predicts correlated signatures in independent probes including.

- baryon acoustic oscillations,
- Type Ia supernovae,
- weak gravitational lensing,
- strong lensing,
- growth of large-scale structure,



- cosmic microwave background,
- primordial nucleosynthesis.

Failure to reproduce these observations simultaneously would falsify the proposed mechanism.

### Prediction 5

The two saturation scales obtained previously,

$$34.6 \text{ Gpc} \quad (61)$$

$$18.2 \text{ Gpc} \quad (62)$$

must produce distinguishable cosmological evolution. Future observational analyses should therefore determine which, if either, corresponds to physical reality.

## Discussion

The principal contribution of this work is not the proposal of a new cosmological model, but the formulation of a coherent phenomenological framework connecting finite holographic capacity,  $\rightarrow$  local representability  $\rightarrow$ , effective gravitational response.

Throughout the manuscript, we have distinguished between mathematical definitions, physical assumptions, derived relations, and speculative interpretation, addressing one of the principal concerns raised during peer review.

Most importantly, the framework preserves microscopic quantum unitarity: only the operational accessibility of information is assumed to become incomplete as causal horizons approach finite capacity. No information is destroyed; part of the physical description is hypothesized to migrate from local semiclassical reconstruction toward globally encoded quantum correlations.

Whether this mechanism contributes to cosmological observations remains an open scientific question. Its validity depends entirely upon future microscopic derivations and confrontation with independent data.

## Conclusion

This work has introduced a phenomenological framework that explores the possible cosmological consequences of finite information capacity associated with causal horizons. The proposal is motivated by the convergence of several well-established theoretical developments, including black-hole thermodynamics, the holographic principle, entanglement entropy, modular Hamiltonians, and emergent approaches to gravitational dynamics.

Rather than modifying quantum mechanics or General Relativity, the framework assumes that finite horizon capacity may limit the amount of information that admits complete local semiclassical reconstruction while preserving global

unitary evolution.

Within this approach, an effective entropy-area response density was introduced,

$$\Sigma(x) = \frac{\delta S_R}{\delta A(x)},$$

Together with a phenomenological saturation criterion motivated by the Bekenstein-Hawking entropy law.

A geometric-informational partition was then proposed,

$$D_{\text{rep}} = \sqrt{\frac{\pi}{6}}, \quad D_{\text{grav}} = 1 - \sqrt{\frac{\pi}{6}},$$

which should be regarded as a normalization hypothesis rather than as a mathematical consequence of Euclidean geometry.

The framework was subsequently connected with the observational discrepancy between early- and late-Universe determinations of the Hubble constant through the effective density parameter.

$$\sigma_{\text{obs}} = \left( \frac{H_0^{\text{late}}}{H_0^{\text{early}}} \right)^2 - 1 \simeq 0.173.$$

A general phenomenological correction to the Friedmann equation was introduced through an undetermined response function.

$$\Phi = \Phi \left( \frac{\Sigma}{\Sigma_{\text{crit}}}, D_{\text{grav}}, \frac{R_H}{R_{\text{lim}}}, z \right),$$

whose derivation from microscopic horizon dynamics remains the principal theoretical objective of the program.

Two independent saturation parameterizations were developed. The first corresponds to pure holographic saturation, whereas the second incorporates the proposed representability weighting. These alternatives predict limiting scales of approximately

$$34.6 \text{ Gpc and } 18.2 \text{ Gpc},$$

respectively. These scales are not presented as observational determinations but as phenomenological consequences of two different normalization assumptions.

The framework further generates several quantitative predictions concerning the expansion history, baryon acoustic oscillations, Type Ia supernovae, weak and strong gravitational lensing, primordial nucleosynthesis, large-scale structure formation, and cosmic microwave background observations. Accordingly, the proposal is experimentally falsifiable: failure to reproduce these independent cosmological constraints simultaneously would rule out the present mechanism, whereas successful agreement would provide evidence supporting the finite-capacity interpretation developed here.

The principal contribution of this work is therefore the formulation of a mathematically explicit and internally consistent research program connecting.



finite horizon capacity –! local representability –! effective cosmological corrections.

Future work will focus on deriving the effective response from microscopic quantum-information dynamics, extending the formalism beyond idealized spherical horizons, and confronting its predictions with precision cosmological observations.

## Declarations

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**Conflicts of interest:** The author declares that there are no conflicts of interest.

**Data availability:** No new observational datasets were generated or analyzed in this theoretical work.

**Code availability:** No computational code was required for the present study.

## Author contributions

The author developed the conceptual framework, mathematical formulation, theoretical analysis, and manuscript preparation.

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