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Research Article

Observer in Quantum Cosmology

Natalia Gorobey[1], Alexander Lukyanenko[1] and Alexander V Goltsev[2]*

[1]Peter the Great Saint Petersburg Polytechnic University, Polytekhnicheskaya 29, 195251, St. Petersburg, Russia

[2]Department of Physics, I3N, University of Aveiro, 3810-193 Aveiro, Portugal

Abstract

A mechanism for the decoherence of the quantum state of the universe is proposed. It reduces to local observation in a certain region, which we call the "observer." The observer differs from the rest of the universe by an additional condition—the Noether identity for the energy-momentum tensor of matter. This additional condition is introduced into gravity theory using undetermined Lagrange multipliers, which act as new independent variables in the region of observation. This modification does not violate the covariance of the theory and also implies a covariant quantization method. The quantum principle of least action, based on the generalized canonical De Donder-Weyl representation, is proposed as such. The real part of the eigenvalue of the De Donder-Weyl action operator is proposed to be considered as an action functional for the quantum universe with an observer.

More Information

***Corresponding author:** Alexander V Goltsev, Department of Physics, I3N, University of Aveiro, 3810-193 Aveiro, Portugal, Email: goltsev@ua.pt

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Introduction

The question of observation or measurement in quantum cosmology may seem far-fetched at a time when the problem of measurements in quantum mechanics remains unsolved. However, the example of Everett's many-worlds interpretation of quantum mechanics [1] shows us the opposite: perhaps it is precisely at the cosmological level that we should seek a solution to the problems of local measurements. The problem is essentially one: how does the destruction of the quantum superposition of potential possibilities occur during measurement? Here, of course, the focus remains on local mechanisms of decoherence as a result of the continuous process of interaction of the quantum system with the environment [2-4]. This is important in connection with quantum computing, whose capabilities are clearly limited by decoherence. Mitigation the decoherence by periodically measuring ancilla qubits after entanglement to reveal error information, allowing a quantum system to detect and correct faults in real time without collapsing the encoded logical qubit [5]. Interest in the phenomenon of decoherence is not limited to problems of quantum computing. The question of the role of the observer and their consciousness in the process of decoherence also arises [6,7]. What role can quantum cosmology play in this? Any interaction, including measurement, is reduced to the exchange of energy and momentum between bodies, and this constitutes the content of the fundamental laws of motion of matter, which are contained in the covariant law of conservation of the energy-momentum tensor:

$$\nabla_\mu T^{\mu\nu} = 0. \quad (1)$$

It should be emphasized that the tensor conservation law (1) is not simply part of the equations of motion of matter. It is an identical consequence of Einstein's equations for geometry, which in turn reflects a deep connection between symmetries (in this case, general covariance) and conservation laws (Noether's theorems [8,9]). As for decoherence, we again refer to Everett's many-worlds interpretation, where all potential possibilities are realized in different branches of cosmological evolution, although without specifying a specific mechanism for this "branching". In this paper, identity (1) is proposed as a mechanism for cosmological decoherence as an additional condition when quantizing the theory of gravity in some region of space-time. We call such a region the region of observation or "observer". Here we follow the approach outlined in [4], but we consider it important to take into account the covariance when modifying the integration measure in the functional integral, which is achieved by the additional condition (1). The additional condition (1) assumes the presence of time in the quantum theory of gravity (QTG) and the evolution of the universe from some initial state. This is not present in the "frozen" formalism of QTG, based on the Wheeler-

DeWitt equation [10]. Time is also absent in the known scenarios of the birth of the universe from “nothing” by tunneling [11] and the no-boundary wave function [12]. In the works [13,14] the problem of time is discussed in connection with the concept of the proper mass of the universe, and a formulation of the QTG is proposed, based on the quantum principle of least action (QPLA). The formalism of the QPLA is equally suitable for formulating the dynamics and for constructing a theory of the initial state of the universe in its Euclidean sector. In this paper, we use a new formulation of the QTG to construct a theory of local observation in quantum cosmology.

In the next section we formulate the QPLA, starting with the simplest mechanical system. In the third section we introduce a generalized canonical form of gravity theory, in which time and spatial coordinates are equivalent. In the fourth section we formulate the QPLA in the generalized canonical form of gravity theory. The QTG in generalized canonical form is considered in Section 4. The possible initial state of the universe is discussed within the framework of QPLA in Section 5. The observation region in the quantum universe is explicitly introduced for the first time in Section 6. Section 7 formulates a decoherence mechanism in which the observer’s state is one-to-one related to the future state of the universe.

Quantum principle of least action

Let us formulate the QPLA first for the simplest mechanical system (particle) with one coordinate q, the Hamiltonian function of which is

$$H(p,q)=\frac{p^2}{2}+V(q,t). \tag{2}$$

The evolution of the particle’s state over time is determined by the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=H(\hat{p},q,t)\psi \tag{3}$$

An alternative formulation of quantum mechanics can be obtained as follows. Let $\psi(t,q)$ be a solution of the equation (3) with the initial condition $\psi(0,q)$. Take an arbitrary trajectory of a particle $q = q(t)$ on a time interval $t\in[0,T]$ and approximate it by a broken line with vertices $q_k=q(t_k)$, where $t_k=k\epsilon, k=1,2,\ldots,N, \epsilon=T/N$. Consider the product

$$\Psi[q_k]=\prod_k\psi(t_k,q_k) \tag{4}$$

This is a function of many variables q_k. In the limit N→∞ we obtain the wave functional $\Psi[q(t)]$ defined on the trajectories of the particle $q = q(t)$ in the configuration space. We introduce the variational derivative of this functional using its multiplicative approximation (4) as follows [15]:

$$\frac{\delta\Psi}{\delta_t q(t_k)}\equiv\frac{1}{\epsilon}\frac{\partial\Psi[q_k]}{\partial q_k}. \tag{5}$$

the notation δ_t indicates that the time function, which is the argument of $\Psi[q(t)]$, is varied. In the new formulation of quantum mechanics, we realize the canonical momentum as an operator of functional differentiation on the space of wave functionals:

$$\hat{p}(t)\equiv\frac{\tilde{\hbar}}{i}\frac{\delta}{\delta_t q(t)}, \tag{6}$$

where, according to (5),

$$\tilde{\hbar}=\hbar\epsilon. \tag{7}$$

Substituting (6) into the canonical form of the particle action, we obtain the action operator

$$\hat{I}_q=\int_0^T dt\left[\dot{q}(t)\frac{\tilde{\hbar}}{i}\frac{\delta}{\delta_t q(t)}+\frac{\tilde{\hbar}^2}{2}\frac{\delta^2}{\delta_t q(t)^2}-V(q,t)\right] \tag{8}$$

on the space of wave functionals $\Psi[q(t)]$. The QPLA is formulated as a secular equation for this operator:

$$\hat{I}_q\psi=\Lambda\psi \tag{9}$$

The assertion is that the multiplicative wave functional (4) is an eigenfunction of the action operator if $\psi(t_k,q_k)$ is a solution of the Schrödinger equation at time t_k. In this case, the eigenvalue is:

$$\text{Ë}=\frac{\hbar}{i}\left[ln\psi(T,q_T)-ln\psi(0,q_0)\right]. \tag{10}$$

This is easy to verify by substituting (4) into (9) (also replacing the integral with an integral sum), and using the exponential form of the wave function:

$$\psi(t,q)=exp\left[\frac{i}{\hbar}\chi(t,q)\right]. \quad (11)$$

Note that the wave functional, taking into account (11), can also be represented in exponential form in the limit ϵ→0:

$$\psi\left[q(t)\right]=exp\left[\frac{i}{\tilde{\hbar}}\int_0^T dt\chi\left(t,q(t)\right)\right]. \quad (12)$$

This representation of the wave functional is singular as ϵ→0. However, it is now completely replaced by the local function χ(t,q(t)), and the first variational derivative is defined as:

$$\frac{\tilde{\hbar}}{i}\frac{\delta\psi}{\delta_t q(t)}=\frac{\partial\chi\left(t,q(t)\right)}{\partial q}\psi. \quad (13)$$

The only difficulty that may arise when substituting (12) into (9) is the calculation of the second variational derivative in (8). However, we have:

$$\tilde{\hbar}^2\frac{\delta^2\psi}{\delta_t q(t)^2}=-\frac{\tilde{\hbar}}{i}\frac{\delta}{\delta_t q(t)}\left[\frac{\partial\chi\left(t,q(t)\right)}{\partial q}\psi\right]=-\left(\frac{\partial\chi\left(t,q(t)\right)}{\partial q}\right)^2\psi$$

$$-\frac{\tilde{\hbar}}{i}\frac{\delta}{\delta q(t)}\int\delta(t-s)\frac{\partial\chi\left(s,q(s)\right)}{\partial q}ds\psi=-\left[\left(\frac{\partial\chi\left(t,q(t)\right)}{\partial q}\right)^2+\psi\frac{\hbar}{i}\epsilon\delta(0)\frac{\partial^2\chi(t,q)}{\partial q^2}\right]\psi. \quad (14)$$

In the discrete approximation ϵδ(0)=ϵ(1/ϵ)=1, so the second variational derivative is defined. The first term under the integral sign in (8) gives:

$$\dot{q}(t)\frac{\tilde{\hbar}}{i}\frac{\delta\psi}{\delta_t q(t)}=\dot{q}(t)\frac{\partial\chi}{\partial q}\psi=\left[\frac{d\chi}{\mathrm{dt}}-\frac{\partial\chi}{\partial\mathrm{t}}\right]\psi. \quad (15)$$

Putting it all together, we obtain: the eigenvalue Λ in (9) does not depend on the internal points of the particle trajectory q(t) and is equal to (10) if the wave function (11) obeys the Schrödinger equation (3).

The formulation of the QPLA for this simplest mechanical problem is easily transferred to QTG. For a given initial state ψ_{in}, the evolution of the universe is determined by the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=\int_\Sigma d^3x\left(N\hat{\mathcal{H}}+N_i\hat{\mathcal{H}}^i\right)\psi, \quad (16)$$

where *N, N_i* are the Arnowitt, Deser, and Misner (ADM) lapse and shift functions [16], and $\hat{\mathcal{H}},\hat{\mathcal{H}}$ are the constraint operators. We write the classical ADM constraints for the simplest case, when matter consists of one real scalar field:

$$\mathcal{H}=\frac{\kappa}{\sqrt{g}}\left(Tr\pi^2-\frac{1}{2}(Tr\pi)^2\right)-\frac{1}{\kappa}\sqrt{g}R-\sqrt{g}\left[\frac{p_\varphi^2}{2g}+\frac{1}{2}g^{ik}\partial_i\varphi\partial_k\varphi+V\right], \quad (17)$$

$$\mathcal{H}^i=-2\pi^k_{|k}-p_\varphi g^{ik}\partial_k\varphi. \quad (18)$$

Here π^{ij} and p_φ are the momentum densities canonically conjugate to the 3D metric g_{ij} on the spatial section Σ and the scalar field, $\Psi\left[g_{i\nu}(x^i),\varphi(x^i)\right]$ (velocity of light c = 1). Canonical quantization is reduced to replacing the canonical momenta with operators,

$$\hat{\pi}^{ij}\left(x^k\right)=\frac{\hbar}{i}\frac{\delta}{\delta g_{ij}\left(x^k\right)},\ \hat{p}_\varphi\left(x^k\right)=\frac{\hbar}{i}\frac{\delta}{\delta\varphi\left(x^k\right)}, \quad (19)$$

and substituting them into the gravitational connections (17), (18). Here δ is the usual variation of the function of three spatial variables. The problem of ordering non-commuting factors in (16) (see [17,18]) is not discussed here.

The initial state of the universe ψ_{in} is not, as we expect, a solution of the WDW equation, and the wave function ψ depends explicitly on time. The kernel of the evolution operator for the Schrödinger equation (16) can be represented by a functional integral in which integration over the functions *N, N_i* is not performed. This evolution is unitary if the constraint operators in (16) are Hermitian. In the transition to the QPLA formalism, the wave functional of the universe is defined on the world history of 4D geometry and matter fields: $\Psi\left[g_{i\nu}\left(x^\mu\right),\varphi\left(x^\mu\right)\right]$. Just like the classical action, the action operator and its eigenvector Ψ must be invariants of generally covariant transformations. Arbitrary lapse and shift functions *N, N_i* ensure this invariance in the same way as in classical gravity theory. Let us represent the wave functional in exponential form:

$$\psi\left[g_{i\nu}\left(x^\mu\right),\varphi\left(x^\mu\right)\right]=\exp\left\{\frac{i}{\tilde{\hbar}}\int_0^{T_0}dx^0\chi^0\left(x^0,g_{i\nu}\left(x^0,\left[x^k\right],\varphi\right)\left(x^0,\left[x^k\right]\right)\right)\right\}. \quad (20)$$

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The notations indicate that the fields enter X^0 as functions of time $x^0 = t$, , but X^0 is a functional of the same fields as functions of spatial coordinates. The function X^0 is the logarithm of the wave function of the universe – the solution of the Schrödinger equation (16) in the direction of the time coordinate x^0. Formula (20) is the first step towards a more symmetrical representation of QTG with respect to space-time coordinates.

Generalized canonical form of action of the theory of gravity

In the standard canonical form of dynamical theory, the isolation of the time parameter formally violates covariance already at the classical level. However, there is a generalization of the canonical representation proposed by De Donder [19] and Weyl [20] that eliminates this violation. In this paper we will restrict our consideration to the simplest set of matter fields consisting of one or more scalar fields, and therefore we will begin our consideration with the action of a real scalar field in Lagrangian form (metric signature (+,-,-,-))

$$I_\varphi = \int \sqrt{-g}\, d^4x \left[\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right]. \tag{21}$$

Let us introduce the generalized canonical DeDonder-Weyl (DDW) momenta:

$$p^\mu = \frac{\partial \mathcal{L}_\varphi}{\partial(\partial_\mu\varphi)} = \sqrt{-g}\, g^{\mu\nu}\partial_\nu\varphi, \tag{22}$$

and also define the generalized density of the Hamiltonian function of a scalar field:

$$\mathcal{H}_\varphi \equiv p^\mu\partial_\mu\varphi - \mathcal{L}_\varphi = \frac{1}{2}\frac{g_{\mu\nu}p^\mu p^\nu}{\sqrt{-g}} + \sqrt{-g}V. \tag{23}$$

After this, we write the action (21) in the generalized canonical form of the DDW:

$$I_\varphi = \int d^4x \left[\partial_\mu\varphi p^\mu - \mathcal{H}_\varphi\right]. \tag{24}$$

Let's do the same with the Hilbert-Einstein action,

$$I_{HE} = -\frac{1}{\kappa}\int d^4x\sqrt{-g}R, , \tag{25}$$

Its Lagrangian density can be represented as [21]:

$$\kappa\mathcal{L}_{HE} = -\sqrt{-g}R = -\partial_\mu\left(\sqrt{-g}\tilde{\Gamma}^\mu\right) - \sqrt{-g}G, \tag{26}$$

where

$$\tilde{\Gamma}^\mu = g^{\alpha\beta}\tilde{\Gamma}^\mu_{\alpha\beta} - g^{\alpha\mu}\tilde{\Gamma}^\beta_{\alpha\beta}, \tag{27}$$

$$G = g^{\mu\nu}\left(\tilde{\Gamma}^\alpha_{\mu\beta}\tilde{\Gamma}^\beta_{\nu\alpha} - \tilde{\Gamma}^\alpha_{\mu\nu}\tilde{\Gamma}^\beta_{\alpha\beta}\right), \tag{28}$$

and

$$\Gamma_{a|bc} = \frac{1}{2}(\partial_b g_{ac} + \partial_c g_{ab} - \partial_a g_{bc}) \tag{29}$$

- Christoffel symbols (the indices are raised and lowered using the metric tensor). The first term on the right-hand side of (26) will be important for formulating the boundary conditions at the corresponding stages of the consideration. Now, to determine the generalized canonical momenta of the gravitational field, it is convenient to start with the momenta conjugate to the Christoffel symbols, which we define as follows:

$$p^{a|bc} \equiv \frac{\partial\left(-\sqrt{-g}G/\kappa\right)}{\partial\tilde{\Gamma}_{a|bc}} = -\frac{\sqrt{-g}}{\kappa}\left[\tilde{\Gamma}^{c|ba} + \tilde{\Gamma}^{b|ca} - g^{bc}\tilde{\Gamma}^{m|a}{}_m - \frac{1}{2}\left(g^{ca}\tilde{\Gamma}^{b|m}{}_m + g^{ba}\tilde{\Gamma}^{c|m}{}_m\right)\right]. \tag{30}$$

Just like $\Gamma_{a|bc}$ the momenta $p^{a|bc}$ are symmetrical with respect to the pair of indices *bc* By folding equality (30) in two different ways, we find,

$$\frac{\sqrt{-g}}{\kappa}\tilde{\Gamma}^{a|m}{}_m = \frac{2}{3}p^{m|a}{}_m, \tag{31}$$

$$\frac{\sqrt{-g}}{\kappa}\tilde{\Gamma}^{m|a}{}_m = -\frac{1}{2}p^{a|m}{}_m + \frac{1}{3}p^{m|a}{}_m. \tag{32}$$

We also have:

$$-\frac{\sqrt{-g}}{\kappa}\left(\tilde{\Gamma}^{c|ba} + \tilde{\Gamma}^{b|ca}\right) = -\frac{\sqrt{-g}}{\kappa}\sqrt{-g}\partial^a g^{bc} = p^{a|bc} + \frac{1}{2}g^{bc}p^{a|m}{}_m + \frac{1}{3}\left(g^{ac}p^{m|b}{}_m + g^{ab}p^{m|c}{}_m - g^{bc}p^{m|a}{}_m\right). \tag{33}$$

Now we introduce the generalized canonical momenta of the DDW $\pi^{a|bc}$, conjugate to g_{bc}, which we find from the equality,

$$p^{a|bc}\tilde{\text{A}}_{a|bc} = \pi^{a|bc}\partial_a g_{bc}. \tag{34}$$

We find:

$$\pi^{a|bc} = \frac{1}{2}\left(p^{b|ac} + p^{b|ca} - p^{a|bc}\right), \tag{35}$$

$$p^{c|ab} = \pi^{a|bc} + \pi^{b|ca}. \tag{36}$$

After this, we write the Hilbert-Einstein action in the generalized canonical form of the DDW:

$$I_{HE} = \int d^4x\left[\partial_a g_{bc}\pi^{a|bc} - \mathcal{H}_g\right], \tag{37}$$

where the density of the generalized Hamiltonian function is equal to

$$\mathcal{H}_g = \frac{\kappa}{\sqrt{-g}}\left[\pi_{a|bc}\pi^{b|ac} + \frac{1}{3}\pi_a^{\ |m}{}_m\pi^{n|a}_{\ \ |\ n} + \frac{1}{3}\pi^{m|}_{\ \ |am}\pi^{n|a}_{\ \ |\ n} - \frac{1}{6}\pi_a^{\ |m}{}_m\pi^{a|n}_{\ \ |\ n}\right]. \tag{38}$$

Quantum theory of gravity in generalized canonical form

The papers [22] and [23] discuss a form of the symplectic structure of gravity that is equivalent to the De Donder-Weyl formalism and can be used for quantization. We can solve the problem of quantizing the theory of gravity in a generalized canonical form within the framework of the QPLA formalism. Let's start with a real scalar field. The usual quantization procedure implies the selection of the "time" parameter by 3+1 partitioning of 4D space. In the covariant theory, this parameter can be any coordinate x^a, $a = 1,2,3,4.$. The three remaining coordinates will be denoted by $x^{\bar{a}}$. We will also introduce a rectangular lattice with elementary translation vectors $\in^a$ in 4D space. We will consider the scalar field $\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)$ as a mechanical system with an infinite set of degrees of freedom, which are numbered by the 3D index $x^{\bar{a}}$, and moving in "time" x^a. We implement the components of the generalized operator of the momentum of the scalar field on the space of wave functionals Ψ[φ(x)] as follows. We represent the wave functional in exponential form (there is no summation over the index a)

$$\varnothing\left[\varphi(x)\right] = exp\left[\frac{i}{\tilde{\hbar}_a}\int_0^{T_a} dx^a\chi^a\left(x^a,\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)\right)\right], \tag{39}$$

where

$$\tilde{\hbar}_a = \hbar\epsilon^a. \tag{40}$$

Then by definition:

$$\hat{p}^a(x)\Psi \equiv \frac{\delta\chi^a\left(x^a,\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)\right)}{\delta\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)}\Psi. \tag{41}$$

Let us emphasize once again that here the variational derivative is taken with respect to the field φ as a 3D functional in the space of indices $x^{\bar{a}}$. Thus, we now have four functions $\chi^a\left(x^a,\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)\right)$, which represent one wave functional (39) for each possible choice of the time parameter. They are not independent, since from (39) it follows:

$$\int_0^{T_a} dx^a\chi^a \,/ \int_0^{T_b} dx^b\chi^b = \epsilon^a/\epsilon^b, \tag{42}$$

where this relation remains finite in the continuum limit. We will also need a representation of the square of the momentum operator. By analogy with (14) we have:

$$\hat{p}^a(x)\hat{p}^b(x)\Psi = \hat{p}^a(x)\left[\frac{\delta\chi^b\left(x^b,\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)\right)}{\delta\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)}\Psi\right] = \frac{\tilde{\hbar}^a}{i}\frac{\delta}{\delta\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)}\left[\frac{\delta\chi^b\left(x^b,\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)\right)}{\delta\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)}\Psi\right]$$

$$= \left[\frac{\delta\chi^a\left(x^a,\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)\right)}{\delta\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)}\frac{\delta\chi^b\left(x^b,\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)\right)}{\delta\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)} + \frac{\hbar}{i}\frac{\delta^2\chi^b\left(x^b,\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)\right)}{\delta\varphi\left(x^a,\left[x^{\bar{a}}\right]\right)\delta\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)}\right]\Psi. \tag{43}$$

The singularity δ(0) here arose (and was compensated by multiplication by $\in^a$) in the second term due to the fact that the first variational derivative $\delta\chi^b\left(x^b,\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)\right)/\delta\varphi\left(x^b,\left[x^{\bar{b}}\right]\right)$ is a function of the coordinate x^a, which is contained among $x^{\bar{b}}$ (if a≠b). The momentum operators of the gravitational field act in the same way. Thus, the generalized Hamiltonian operators $\mathcal{P}^{\nu a}$ and $\hat{\mathcal{H}}_\varphi$ are defined (if we also agree to place the momentum operators on the right in all terms).

Now let us consider the terms $\partial_a g_{bc}\hat{\pi}^{a/bc}$ and $\partial_a \varphi \hat{p}^a$ under the action integral sign (here the summation is performed over repeating indices):

$$\left(\partial_a g_{bc}\hat{\pi}^{a/bc} + \partial_a \varphi \hat{p}^a\right)\text{Ø} = \left[\partial_a g_{bc}\frac{\delta\chi^a\left(x^a, g_{\alpha\beta}\left(x^a,\left[x^{\breve{a}}\right]\right),\varphi\left(x^a,\left[x^{\breve{a}}\right]\right)\right)}{\delta g_{bc}\left(x^a,\left[x^{\breve{a}}\right]\right)}\right.$$

$$\left.+\partial_a\varphi\frac{\delta\chi^a\left(x^a, g_{\alpha\beta}\left(x^a,\left[x^{\breve{a}}\right]\right),\varphi\left(x^a,\left[x^{\breve{a}}\right]\right)\right)}{\delta\varphi\left(x^a,\left[x^{\breve{a}}\right]\right)}\right]\Psi = \left(d_a\chi^a - \partial_a\chi^a\right)\Psi, \tag{44}$$

where $d_a\chi^a$ is the sum of the total derivatives of the components of the wave function with respect to the "times" x^a, and $d_a\chi^a$ is the corresponding sum of the partial derivatives.

We are ready to formulate the QPLA in an arbitrary domain Ω and derive the required consequences from it. Now the four components of the wave function also depend on the Euclidean metric: $\chi^a\left(x^a, g_{\alpha\beta}\left(x^a,\left[x^{\breve{a}}\right]\right),\varphi\left(x^a,\left[x^{\breve{a}}\right]\right)\right)$. The action operator has the form:

$$\hat{I}_\Omega = \int_\Omega d^4x\left[\partial_a g_{bc}\hat{\pi}^{a|bc} + \partial_a\varphi\hat{p}^a - \hat{\mathcal{H}}_g - \hat{\mathcal{H}}_\varphi\right]. \tag{45}$$

The boundary contribution to the action will be considered separately in each case. We are interested in the eigenvalue of the action operator, which can be written in the form

$$\Lambda = \frac{\hat{\mathrm{I}}\Psi}{\Psi}. \tag{46}$$

By definition, Λ does not depend on the values of the metric tensor and the matter fields inside the domain Ω, but only on their boundary values on ∂Ω. The integral of the total derivative in (44) gives the desired eigenvalue

$$\Lambda = \oint_{\partial\Omega} dSn_a\chi^a, \tag{47}$$

n_a is the unit vector of the outward normal to the surface ∂Ω. If the domain Ω is a rectangular parallelepiped, then the integral (45) is reduced to the sum of the differences $\chi^a(T_a) - \chi^a(0)$ on the opposite faces. Thus, the eigenvalue of the action operator is reduced to the algebraic sum (integral) of the boundary values of the components x^a of the wave function, provided that the volumetric part of the action vanishes for any configuration of geometry and matter fields inside Ω. This condition reduces to a variational-differential equation for the components of the wave function:

$$\partial_a\chi^a\Psi + \left(\hat{\mathcal{H}}_g + \hat{\mathcal{H}}_\varphi\right)\Psi = 0. \tag{48}$$

The wave functional Ψ in this equation is a common factor due to (41) and (43) and can be reduced. But we leave it, keeping in mind some complication of the structure of the action in the observation area. The nonlinear equation (48) with respect to the components x^a of the wave function should be supplemented by proportions (42). Equation (48) replaces the Schrödinger equation (16) in the new formulation of QTG.

Initial state of the universe

Since we need time in QTG and the real dependence of the state of the universe on time, we will also need the initial state of the universe. The description of evolution in the formalism of QPLA is invariant with respect to arbitrary transformations of space-time coordinates, if they do not affect the initial and final spatial sections. Invariance is expressed in the fact that the eigenvalue of the action operator (by definition) does not depend on the internal points of the world history of geometry and matter fields. In the works [13,14], the QPLA in the generalized canonical form of the DDW was used to formulate the quantum dynamics of the universe in the "polar" region centered at the "South Pole" of the no-boundary Hartle-Hawking wave function [12]. The boundary of the polar region – the "polar circle" – is, according to our assumption, a spatial section of the universe Σ_{in}, beyond which evolution in real time begins. In the polar region itself, the Euclidean form of the QPLA with imaginary time is used. Thus, the difference between time and space coordinates completely disappears, and there is no need to connect the singular point $g = \det g_{\mu\nu} = 0$ with the pole of the spherical coordinate system in which the radii are the lines of time [12]. In this case, there is no need for any boundary conditions at the pole. In order for such a formulation of the QPLA to provide the necessary initial state of the universe Ψ_{in} on Σ_{in}, it is necessary to justify the existence of the corresponding classical solution in the polar region – the instanton. In [13,14], a justification was proposed for the 3D scale factor of the universe $g^{(3)} = det g_{ij}(x)$. It consists in the fact that for the dynamics of this variable it is possible to prove an analogue of the theorem of positivity of energy, based on the Witten identity [24], if the square of the eigenvalue of the 3D Dirac operator on a spatial section is taken as the expansion energy. In this section we consider such a justification for the dynamics of the conformal factor K of the 4D metric, $g_{ab} = \hat{E}^2 B_{ab}$, where B_{ab} is the base metric with $detB_{ab} = 1$ [25]. The Hilbert-Einstein Euclidean action in the new variables takes the form:

$$I_{HE} = -\frac{1}{\kappa}\int_\Omega d^4x \left(K^2 R_B + 6B^{ab}\partial_a K \partial_b K\right), \tag{49}$$

where

$$R_B = \partial_a\left(B^{bc}\mathcal{G}^a{}_{bc}\right) + B^{ab}\mathcal{G}^c{}_{ad}\mathcal{G}^d{}_{bc}, \tag{50}$$

is the Ricci scalar, and $\mathcal{G}^c{}_{ad}$ are the Christoffel symbols for the base metric B_{ab}. We have not written out the surface contributions to the action (49) (see [25]), which must be taken into account in the continuity conditions for the components of the metric tensor g_{ab} on the boundary $\partial\Omega = \acute{O}_{in}$ are interested in the dynamics of the conformal factor K in the domain Ω. To simplify the consideration, we choose in Ω harmonic coordinates [26]:

$$B^{bc}\mathcal{G}^a{}_{bc} = -\partial_b B^{ab} = 0. \tag{51}$$

Now we represent the remaining expression in (50) as the difference of two positive definite contributions:

$$R_B = B^{ab}B^{cp}B^{dq}\mathcal{G}_{p|ad}\mathcal{G}_{q|bc} = B^{ab}B^{cp}B^{dq}\mathcal{G}_{(p|a)d}\mathcal{G}_{(q|b)c} - B^{ab}B^{cp}B^{dq}\mathcal{G}_{[p|a]d}\mathcal{G}_{[q|b]c}c\,, \tag{52}$$

where the round brackets denote symmetrization by a pair of indices, and the square brackets denote antisymmetrization. The first of them corresponds to the longitudinal components of the gravitational field (together with the conformal factor), and the second to the transverse components associated with gravitational waves. Thus, the density of the Lagrange function of the theory of gravity after the selection of the conformal factor can be written as:

$$\mathcal{L}_g = -\frac{1}{\kappa}\left(6B^{ab}\partial_a K\partial_b K + K^2 B^{ab}B^{cp}B^{dq}\mathcal{G}_{(p|a)d}\mathcal{G}_{(q|b)c}\right) + \frac{1}{\kappa}K^2 B^{ab}B^{cp}B^{dq}\mathcal{G}_{[p|a]d}\mathcal{G}_{[q|b]c} + K^4\mathcal{L}_m, \tag{53}$$

From the structure of the potential relief for the scale factor it is seen that if the energy of the gravitational waves near the singularity K=0 is less than the longitudinal potential energy, classical motion of K (in imaginary time) from the singular point is possible. This means that there is an instanton in the polar region containing the singularity g=0 inside.

On this basis, it can be expected that the QPLA in the generalized canonical form of DDW

$$\hat{I}_\Omega\Psi = \Lambda_\Omega\Psi \tag{54}$$

has a non-trivial solution in the polar region. To enter the region of motion with a real time parameter, we introduce spherical coordinates in the polar region with a center at an arbitrary point (not associated with the singularity). We pass in the action operator $\hat{I}_\Omega$ to spherical coordinates near the boundary of Ω, where we write it in the usual canonical form with the radial coordinate as the time parameter. The wave functional Ψ in these coordinates can be represented as a product of wave functions defined on surfaces of constant radius. The boundary wave function is equal to

$$\psi_{in} = exp\left[\frac{i}{\hbar}\Lambda\right], \tag{55}$$

provided that $\partial\Omega = \Sigma_{in}$ is defined as the locus of the cusp points of the scale factor:

$$\frac{\delta\Lambda_\Omega}{\delta K} = 0. \tag{56}$$

At this boundary, a Wick rotation should be performed in the complex plane of the radial coordinate. We emphasize once again that the initial state of the universe determined in this way is not a solution of the WDW equation, and the proper mass of the universe in this state is not equal to zero.

The field of observation in the universe

Let us return to the space-time with the Lorentz signature and select some compact, convex region Ω in it. We modify the quantum theory here by the additional condition (1). If we start with the functional integral representing the evolution operator for the Schrödinger equation (16), the modification can be achieved by limiting the measure of the functional integration by the factor [4]

$$\delta^\Omega\left(\nabla_\mu T^{\mu\nu}\right), \tag{51}$$

where δ^Ω is the functional delta function with support Ω. The simple delta function has an integral representation:

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\lambda e^{-i\lambda x}. \tag{52}$$

In our case, the functional delta function (51) will have a similar functional integral representation, in the exponential of which there will be an integral over the domain Ω:

$$-\int_{\Omega}\sqrt{-g}\,d^4x\lambda_\nu\nabla_\mu T^{\mu\nu}. \tag{53}$$

This is equivalent to adding to the classical action of the integral with an arbitrary vector Lagrange multiplier λ_ν. The latter becomes an independent dynamic variable. After integration by parts,

$$\int_{\Omega}\sqrt{-g}\,d^4x\nabla_\mu\lambda_\nu T^{\mu\nu}-\oint_{\partial\grave{U}}dS\sqrt{-g}\,n_\mu\lambda_\nu T^{\mu\nu}. \tag{54}$$

the Lagrange multiplier will be included in the dynamic variables. The surface integral over the boundary of the observation region will be compensated after the main part of the action is reduced to a generalized canonical form. Thus, the density of the Lagrange function of the modified theory has the form:

$$\tilde{\mathcal{L}}=\left(\mathcal{L}_g+\mathcal{L}_\varphi\right)+\sqrt{-g}\nabla_\mu\lambda_\nu T^{\mu\nu}, \tag{55}$$

where

$$\mathcal{L}_\varphi=\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi_i\partial_\nu\varphi_i-V\right], \tag{56}$$

is the density of the Lagrange function of the multiplet of scalar fields of matter, and

$$T^{\mu\nu}=g^{\mu\alpha}g^{\nu\beta}\partial_\alpha\varphi_i\partial_\beta\varphi_i-\frac{g^{\mu\nu}\mathcal{L}_\varphi}{\sqrt{-g}} \tag{57}$$

is the energy-momentum tensor of matter fields. For greater generality, we consider here a multiplet of scalar fields. We define the generalized canonical momenta of the DDW. The generalized momenta for the Lagrange multipliers λ_ν are equal to the density of the energy-momentum tensor:

$$\mathcal{P}^{\mu\nu}=\sqrt{-g}T^{\mu\nu}. \tag{58}$$

The generalized momenta associated with the Christoffel symbols and matter fields will be modified by adding new terms:

$$\tilde{p}^{a|bc}=p^{a|bc}-\lambda^a\mathcal{P}^{bc}, \tag{59}$$

$$\tilde{p}_i^\mu=\sqrt{-g}\left[g^{\mu\nu}+2\nabla_{(a}\lambda_{b)}\mathcal{T}^{ab|\mu\nu}\right]\partial_\nu\varphi_i. \tag{60}$$

where indicated

$$\mathcal{T}^{ab|\mu\nu}=g^{a\mu}g^{b\nu}-\frac{1}{2}g^{ab}g^{\mu\nu}. \tag{61}$$

In order to express, as usual, the generalized velocities $\partial_\nu\varphi_i$ in terms of the generalized momenta $\tilde{p}_i^\mu$, we must use relations (60), where the square bracket contains a symmetric, second-rank tensor $\nabla_{(a}\lambda_{b)}$, which is composed of derivatives and must also be excluded. We will proceed differently. Let us first consider the simplest case, when there is a single real scalar field of matter. The combination of equations (56), (57) and (58) then gives:

$$\mathcal{T}_{\mu\nu|ab}^{-1}\left(\frac{\mathcal{P}^{ab}}{\sqrt{-g}}-g^{ab}V\right)=\partial_\mu\varphi\partial_\nu\varphi, \tag{62}$$

where $\mathcal{T}_{\mu\nu|ab}^{-1}$ is a matrix with paired indices, inverse to (61). From here we find:

$$\partial_\mu\varphi=\sqrt{\mathcal{T}_{\mu\mu|ab}^{-1}\left(\frac{\mathcal{P}^{ab}}{\sqrt{-g}}-g^{ab}V(\varphi)\right)}. \tag{63}$$

For given $\mathcal{P}^{ab}(x)$ and $g^{ab}(x)$ these differential equations are first integrals of the classical equation of motion of a scalar field for each possible choice of the "time" parameter. Integrating further (63), we find the increments of coordinates Δx^μ between the cusp points $\partial_\mu\varphi=0$, measured by the dynamics of the scalar field in the corresponding coordinate direction.

Let's complicate the problem: let there be a triplet of scalar fields φ_i, i=1,2,3. Equation (62) will be replaced by the following:

$$\mathcal{T}_{\mu\nu|ab}^{-1}\left(\frac{\mathcal{P}^{ab}}{\sqrt{-g}}-g^{ab}V\right)=\partial_\mu\varphi_i\partial_\nu\varphi_i, \tag{64}$$

where the summation is performed over the index i. In this case, we also extract the square root using the Dirac method [27]. We replace the triplet φ_i with a 2×2 matrix,

$$\hat{\varphi}=\varphi_i\hat{\sigma}^i, \tag{65}$$

where $\hat{\sigma}^i$ are the Pauli matrices, and we write down the formal solution of equations (63):

$$\partial_\mu \hat{\varphi} = \sqrt{\mathcal{T}^{-1}_{\mu\mu|ab}\left(\frac{\mathcal{P}^{ab}}{\sqrt{-g}} - g^{ab}V\right)}, \tag{66}$$

where the expression on the right is multiplied by the identity 2×2 matrix. These matrix equations will acquire a certain meaning below after introducing the components of the wave function. By collapsing both sides of equation (60) with the Pauli matrices, we also write it in matrix form:

$$\hat{\tilde{p}}^\mu = \sqrt{-g}\left[g^{\mu\nu} + 2\nabla_{(a}\lambda_{b)}\mathcal{T}^{ab|\mu\nu}\right]\partial_\nu \hat{\varphi}. \tag{67}$$

Substituting (66) into (67), we obtain a matrix equation in which the components of the symmetric tensor $\nabla_{(a}\lambda_{b)}$ remain unknown. These equations are sufficient to determine the components of $\nabla_{(a}\lambda_{b)}$ as a linear function of the generalized momenta $\hat{\tilde{p}}^\mu$ in matrix form. Thus, $\nabla_{(a}\lambda_{b)}$ should now also be considered as 2×2 matrices.

Completing the modification of quantum theory in the observation domain Ω, we find the generalized density of the Hamiltonian function:

$$\tilde{\mathcal{H}} = \tilde{p}^{a|bc}\,\tilde{\text{A}}_{a|bc} + \tilde{p}_i{}^\mu \partial_\mu \varphi_i + \mathcal{P}^{\mu\nu}\partial_\mu \lambda_\nu - \tilde{\mathcal{L}} = p^{a|bc}\tilde{\text{A}}_{a|bc} - \sqrt{-g}\mathcal{L}_g + \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu} + 2\nabla_{(a}\lambda_{b)}\mathcal{T}^{ab|\mu\nu}\right]\partial_\mu\hat{\varphi}\partial_\nu\hat{\varphi} + \sqrt{-g}V$$

$$= \tilde{\mathcal{H}}_g\left(\tilde{p}^{a|bc} + \lambda^a\mathcal{P}^{bc}\right) + \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu} + 2\nabla_{(a}\lambda_{b)}\mathcal{T}^{ab|\mu\nu}\right]\partial_\mu\hat{\varphi}\partial_\nu\hat{\varphi} + \sqrt{-g}V. \tag{68}$$

In this expression for the density of the Hamiltonian function of the modified theory, the generalized canonical momenta (58), (59) and (60) (taking into account (66) and (67)) are included everywhere. Let us make a new substitution

$$\tilde{p}^{a|bc} + \lambda^a\mathcal{P}^{bc} = \overset{\approx}{p}{}^{a|bc}, \tag{69}$$

and also transform the product:

$$\tilde{p}^{a|bc}\,\tilde{\text{A}}_{a|bc} = \tilde{\pi}^{a|bc}\partial_a g_{bc} = \left(\overset{\approx}{p}{}^{a|bc} - \lambda^a\mathcal{P}^{bc}\right)\tilde{\text{A}}_{a|bc} = \overset{\approx}{\pi}{}^{a|bc}\partial_a g_{bc} - \frac{1}{2}\left(\lambda^b\mathcal{P}^{ac} + \lambda^b\mathcal{P}^{ca} - \lambda^a\mathcal{P}^{bc}\right)\partial_a g_{bc}. \tag{70}$$

After this, the generalized canonical form of action of the modified theory finally takes the form:

$$\tilde{I}_{\grave{U}} = \int_{\grave{U}} d^4x\left[\pi^{a|bc}\partial_a g_{bc} + p_i^\mu\partial_\mu\varphi_i + \mathcal{P}^{\mu\nu}\nabla_\mu\lambda_\nu - \tilde{\mathcal{H}}\right], \tag{71}$$

where

$$\tilde{\mathcal{H}} = \mathcal{H}_g + \hat{p}^\mu\sqrt{\mathcal{T}^{-1}_{\mu\mu|ab}\left(\frac{\mathcal{P}^{ab}}{\sqrt{-g}} - g^{ab}V\right)} + \sqrt{-g}V, \tag{72}$$

$\mathcal{H}_g$ is the density of the generalized Hamiltonian function of the original theory (38). In the final expressions, we have discarded the now unnecessary "tilde" signs at the canonical momenta. Thus, the gravitational part of the action in the observation region has not changed. However, there are two significant features that require separate consideration.

The first feature is the appearance of a term in the action

$$\int_\Omega d^4x\mathcal{P}^{\mu\nu}\nabla_\mu\lambda_\nu = -\int_\Omega d^4x\nabla_\mu\mathcal{P}^{\mu\nu}\lambda_\nu + \oint_{\partial\Omega} dSn_\mu\lambda_\nu\mathcal{P}^{\mu\nu}. \tag{73}$$

After integration by parts, $\partial\Omega_{obs}$. play the role of the usual canonical momenta, to which the four components of $\mathcal{P}^{\nu a}$ are conjugate, if the coordinate x^a is chosen as the time parameter. Since λ_ν is not contained in the modified Hamiltonian function (72), we obtain the classical continuity equations for the energy-momentum density of matter

$$\nabla_\mu\mathcal{P}^{\mu\nu} = 0, \tag{74}$$

which can be solved before quantizing the modified theory. This can be done if the conditions on the boundary of the observation region are specified. We have two boundary contributions: one in formula (73), and the other in formula (54). They cancel each other out due to equality (58). Thus, there are no contributions to the action on the boundary of the observation region, but the question of fixing the boundary conditions for (74) remains open.

The second feature of the modified action is its linear dependence on the canonical momenta of the matter fields p_i^μ and the matrix nature of this dependence. In the simplest case, when there is only one scalar field, this means that in addition to (74), we also have four equations of the form (63), determining the geometric parameters Δx^μ of the observation region according to the classical dynamics of this field. These equations can also be solved before quantization. If there are more matter fields, after quantization we obtain a spinor equation for the components of the wave functional (summation is performed over the index μ),

$$\partial_\mu \hat{\varphi}\begin{pmatrix}\chi_1^a\Psi_1\\ \chi_2^a\Psi_2\end{pmatrix} = p_i^\mu \sqrt{\mathcal{T}_{\mu\mu|ab}^{-1}\left(\frac{\mathcal{P}^{ab}}{\sqrt{-g}} - g^{ab}V\right)}\begin{pmatrix}\chi_1^a\Psi_1\\ \chi_2^a\Psi_2\end{pmatrix}, \tag{75}$$

which is now represented by four products of two-component spinor wave functions

$$\hat{\Psi}_\mu = \prod_n \hat{\Psi}\left(n\epsilon^\mu, q\left(n\epsilon^\mu, \left[x^{\tilde{\mu}}\right]\right)\right) \tag{76}$$

for each choice of the time parameter x^μ. Here n = $1,2,\ldots N_\mu, T_\mu = N_\mu \epsilon^\mu$,q =, $[g_{ii}, \varphi_k]$. The products in (1) are chronologically ordered in the corresponding "time". We still assume that (1) gives different multiplicative representations of the same functional $\hat{\Psi}$, which is a 2x2 matrix. Such a complication of quantum theory in the observation region raises the question of consistency at the boundary $\partial\Omega_{obs}$ with the rest of the universe. Just as on the boundary with the polar region, here we need to set the boundary wave function of the universe. And we will do the same here: if Λ_{obs} is the eigenvalue of the action operator $\hat{I}_{\Omega_{obs}}$ corresponding to the eigenvector psi in the observation region, then as the boundary value of the wave function we will take

$$\psi_{\partial\Omega_{obs}} = exp\left[\frac{i}{\hbar}\Lambda_{obs}\right]. \tag{77}$$

Observer in quantum cosmology

We have spoken about the region of observation with special laws of motion of matter (74) and (75) inside. Let us now call this region "the observer" and formulate the consequences that can be obtained from the QPLA for its cosmological evolution. The operator of action of the universe is additive and consists of two parts:

$$\hat{I} = \hat{\tilde{I}}_\Omega + \hat{I}_{\Omega'} \tag{78}$$

where the first term is obtained by quantizing the modified action (71), and the second is the action (45) for the rest of the universe. In this case, the wave functional remains the same for the entire considered history of the universe, a two-component spinor Ψ[g(x),φ(x), $P^{\mu\nu}$(x)]. The first term in (71) describes the motion of the energy-momentum $P^{\mu\nu}$ inside the observer, as well as the geometry Δx^μ of its boundary, and the full action describes the dynamics of the fundamental degrees of freedom in the entire universe. All this is contained in the secular equation

$$\left(\hat{I}_\Omega + \hat{I}_{\Omega'}\right)\Psi = \Lambda\Psi, \tag{79}$$

The eigenvalue Λ, by definition, is a c -number that depends only on the boundary values of the dynamic variables. What is important to us is what happens on the inner boundary ∂Ω, separating the observer and the rest of the universe. On it, the fundamental degrees of freedom must be continuous. In addition, to exclude the boundary contribution arising from the total derivative in (26), the normal components (27) must also be continuous. Earlier, we also noted the equality of the normal components $n_a\mathcal{P}^{\mu a}(x)$ and $\sqrt{-g}n_a T^{\mu a}$ on the boundary, which is also satisfied by (58). If all this is satisfied, Λ "does not feel" what is happening on the boundary ∂Ω. However, if we fix some admissible values of the normal components $n_a\mathcal{P}^{\mu a}(x)$ on the boundary, we constrain the set of trajectories in the configuration space of the universe, on which, by definition, Λ does not depend. And Λ will "feel" these admissible values. What determines the admissible values of $n_a\mathcal{P}^{\mu a}(x)$? If on earlier sections of the boundary ∂Ω the energy-momentum flux is determined only by the previous state of the universe, on later sections the continuity equations (74) inside the observer must be taken into account. In other words, the internal dynamics of the observer imposes restrictions on the admissible boundary values $n_a\mathcal{P}^{\mu a}(x)$, and this is "felt" by Λ.

Thus, the formulation of the QTG in terms of the modified QPLA (77) allows us to introduce into consideration the cosmological mechanism of decoherence, in which a certain state of the observer $\hat{\Psi}_\Omega$ is also accompanied by a certain final state of the universe

$$\psi_{out} = exp\left[\frac{i}{\hbar}\Lambda\left(n_a\mathcal{P}^{\mu a}(x)\right)\right]. \tag{80}$$

The direct mathematical connection between what the observer "sees" ($n_a\mathcal{P}^{\mu a}(x)$ on ∂Ω) and the future state of the universe ψ_{out} can be seen as a justification for the many-worlds interpretation of Everett's quantum mechanics. At the same time, the final state ψ_{out} also has a numerical characteristic: its norm depends on the result of local observation. Indeed, the restriction of the set of admissible trajectories in defining Λ in (77), which is the essence of cosmological decoherence, is equivalent to the restriction of the integration measure in the functional integral. The latter obviously entails a violation of the unitarity of the evolution operator. The latter obviously entails a violation of the unitarity of the evolution operator. The presence of such a numerical characteristic of the final state of the universe as $\|\psi_{out}\left(n_a\mathcal{P}^{\mu a}(x)\right)\|$ leaves some ambiguity in the picture of the "branching" of the universe. If we accept the simultaneous realization of all possible observation results and the corresponding branches of cosmological evolution, how should we interpret the mentioned numbers corresponding to each branch? The probabilistic interpretation does not fit into this picture. To interpret the numerical parameterization of the various branches of the Everett multiverse, we will choose a more suitable one, which is closer to the original Hilbert-Einstein principle of least action: the eigenvalue of the action operator of the universe at the final stage of its evolution ($\Lambda_{out}(n_a\mathcal{P}^{\mu a}(x))$). More precisely, we need its real part, averaged over the final state (78):

$$S_{obs}\left[n_a\mathcal{P}^{\mu a}(x)\right] = Re\Lambda_{out}\left(n_a\mathcal{P}^{\mu a}(x)\right)_{\psi_{out}} \tag{81}$$

Conclusion

The modification of the QPLA, by separating the observation region whose boundary is determined by the internal deterministic dynamics of energy and momentum using equations (74), (75), also assumes certain boundary conditions for this internal dynamics. This deterministic dynamics, introduced as an additional condition in quantum theory, serves as a mechanism of decoherence. The cosmological aspect of decoherence is that the observation domain violates unitarity and reduces cosmological evolution to different states corresponding to the observer's state. In this we see the justification for Everett's picture of the multiverse. However, the dependence of the final state of the universe on the state of the observer obtained in this work allows us to interpret this multiverse as a set of trial states in the cosmological principle of least action. It is the only state (if such exists) that corresponds to the extremum of the norm that is actualized. In this case, the final state norm should be understood as an action functional in quantum cosmology, the extremum of which determines the observed world history of the universe, as seen by the observer. The variational parameters here are the energy and momentum flows $n_a\mathcal{P}^{\mu a}(x)$ (as well as other observables) on the boundary $\partial\Omega_{obs}$. But here we go far beyond the limits of computability [5].

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